ON BIAS OF TESTING MERTON'S MODEL

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ABSTRACT

In the credit risk modeling literature, there are two different viewpoints towards the structural model of Merton. Many empirical tests show that the Merton model overestimates corporate bond prices substantially. However, there are other empirical works defensing that the Merton model is useful in predicting default and performs well in estimating deposit insurance fund. In this paper, we argue that the poor performance of the Merton model may be the consequence of using proxies in empirical studies. Specifically, we show theoretically that using sum of market value of equity and book value of corporate liabilities as a proxy for the market value of corporate assets generates significant bias of overestimating the asset values. It follows that the market value of corporate bond, as a risk-free bond less a put option on corporate asset, would be overestimated under the Merton approach. We propose that the asset values and volatilities are better estimated with maximum liklihood estimation (MLE). To support this claim, we conduct a simulation and an empirical study. Our empirical results document that, if MLE is adopted, the Merton model can overestimate or underestimate the bond prices with the average percentage error of 0.16%.

KEY WORDS

Corporate Bond Pricing, Credit Risk, Merton's Model, MLE

1 Introduction

The seminal works of Black and Scholes (1973) [1] and Merton(1974; henceforth the Merton model) [2] have had a great impact on risky debt pricing. However, the Merton model can deal with zero coupon bonds only. To tackle this shortcoming, Geske (1977) [3] proposed a model to treat a coupon bearing bond as a compound option. Also, the Merton model assumes constant interest rate and ignores the possibility of default prior to the debt maturity. Longstaff and Schwartz (1995) [4] proposed a model which allows stochastic interest rate, partial recovery for bondholders and early default.

In the last few decades, there are several empirical tests on these models. Jones, Mason amd Rosenfeld (1984) [5] implemented extensively a test for the Merton model. They used a sample of firms with simple capital structure and bond prices in secondary market during 1977-1981 period. They found that the predicted prices from the Merton model were too high, about 4.52% overestimation on average. They also found that the errors were more severe for non-investment grade bonds, but it worked quite good for investment grade bonds.

Ogden (1987) [6] conducted another empirical study and found similar results. They found that the yield spreads were underpredicted by about 104 basis points on average when using the Merton model. Eom, Helwege and Huang (2004; henchforth EHH) [7] performed another extensive tests of several structural bond pricing models, including the Merton model. By using market values of corporate bonds with simple capital structure, they found that the Merton model underpredicted credit yield spread, or overpriced corporate bonds prices, significantly.

The concensus is that the Merton model overprices bonds and underpredicts credit yield spreads. In contrary, we argue that the inadequate power of the Merton model may not be due to the construction of the model. Instead, we find some evidence that the consistently underprediction of credit yield spreads is partly (if not solely) due to using a proxy [1] to estimate the market value of firm's assets in empirical studies. Use of proxy to estimate market values of firms' assets may be due to the accounting rule that book value of assets is equal to the sum of book value of equity and liabilities. Hence, one may intuitively adds observable market values of equitiews to book values of liabilities so as to obtain unobservable market values of firms' assets.

We verify our claim theoretically and empirically. In a theoretical perspective, we employ option's properties to show that the proxy must overestimate the market values of the firm's assets. It follows that a put option on the underlying asset is underpriced, implying that the Merton corporate bond price, as a risk-free bond less the put, is overestimated. We propose that the firm value and volatility are better estimated by a maximum liklihood estimation (MLE). In our simulation study, we find that MLE does an excellent job in estimating the two parameters. However, the proxy greatly overstates the firm value. The simulation exercise also shows that the resulting Merton prices of corporate bonds are overestimated for the case of using the

[1] The market value of the firm's assets is estimated as the sum of the market value of equities and book value of toal liabilities

proxy but archives a good prediction for the case of MLE.

Our empirical study further supports the claim. We follow the empirical construction of EHH (2004). That means we select corporate bonds according to their criteria and use their extended Merton model to compute corporate bond prices with recovery rate of 51.31%. The only difference is that we estimate market values of corporate assets with MLE instead of the proxy. Our result is so amazing that the Merton model can overprice or underprice corporate bonds with difference between predicted yield and true yield of 3 basis points on average. Hence, different to previous fellows' conclusions, we do not agree that the Merton model consistently overprices corporate bonds. However, the standard deviation for the percentage error is very large, meaning that the Merton model still has to be improved in many dimensions.

We believe that the model does not work well because there is no such a firm whose capital structure is as simple as the Merton's assumptions. Therefore, the EHH (2004) extended Merton model is indeed a partially wrong approach. Another reason is that it is very difficult to obtain other parameters, like the recovery rate. In this paper, we assume the recovery rate to be a constant of 53.31%, following EHH, which is just an another proxy.

The rest of the paper is organized as follows. Section 2 derives implications of using the proxy by financial arguments. Section 3 shows the simulations results of estimating bond prices, yields and credit yield spreads by the MLE approach and by the proxy approach. Section 4 shows the empirical results. Section 5 concludes this paper.

2 Implications of proxy

This section derives implications of using the proxy under the Merton framework. We find that the proxy leads to the result that market values of corporate assets are implicitly overestimated. After plugging into the model, no matter how accurate the other variables are and what empirical data are observed, corporate bond prices are overpredicted under the Merton model. The followings show that the results of overpricing of corporate bond when using the Merton model and extended Merton model is due to the inappropriate use of the proxy.

2.1 Zero coupon bonds

Merton (1974) proposed a pricing formula for a zero coupon bond. If there is no intermediate default, the terminal payoff of a corporate bond will be the minimum of the promising payments (X) and the market value of assets at that time (V_T). After discounting back and doing some simple calculations, it can be shown that current price of a corporate bond (BP_c) is equal to a risk-free bond, with the same maturity (T) and principal payment (X), minus a put option ($P(V,X,T)$) on the current market value of assets (V) with strike price equal to its principal payment (X) and the same maturity T, i.e.,

$$BP_c(0,T) = X \cdot D(0,T) - P(V,X,T), \qquad (1)$$

where $D(0,T)$ denotes the price of risk-free (default-free) bond at time 0 with principal payment \$1 and maturity T.

In order to use this model, we have to obtain the market value of the firm's asset (V) in advance. As the asset value is not observable, EHH took a proxy that the estimated market value of assets (V_{proxy}) is the sum of total liabilities (X) and market value of equity (V_E), i.e.,

$$V_E = V_{proxy} - X, \qquad (2)$$

Denote V_{true} as the true market value of assets. By using the standard call framework for securities valuation, and applying no arbitrage argument, we know that a call option premium must ahead its intrinsic value. Specifically,

$$C(V_{true}, X, T) = V_E = V_{proxy} - X < C(V_{proxy}, X, T), \qquad (3)$$

Since call option is an increasing function of its underlying asset, which is V in (3), we can see that EHH has effectively overestimated the market value of firm's assets, i.e., $V_{proxy} > V_{true}$. As the market value of assets has been overestimated, the corresponding put option in equation (1) is underestimated as a results. So, the predicted price of corporate bonds will be too high and thus predicted yield spreads will be underestimated. This accounts for the EHH findings for zero coupon bonds.

2.2 Coupon bearing bonds

Since Merton (1974) model can deal with only zero coupon bonds, EHH introduced an extended Merton model for coupon bearing bonds. In fact, they treat a coupon bond as a portfolio of zero coupon bonds. Hence, with similar notation and annual coupon rate c, the formula of a corporate bond paying semiannual coupons can be simplified as

$$\begin{aligned} & BP_c(0,T) \\ = & \sum_{i=1}^{N-1} \left[\frac{Xc}{2} D(0,t_i) - P(\frac{Xc}{2}, V, t_i) \right] \\ + & \left[X(1+\frac{c}{2}) \cdot D(0,T) - P(X(1+\frac{c}{2}), V, T) \right], \end{aligned} \qquad (4)$$

where we have N coupon paying dates of $\{t_1, t_2, \cdots, t_N\}$ and $t_N = T$.

Once again, if we use the overstated market value of assets, the corresponding put options in equation (4) will be underestimated. This makes the predicted price of corporate bonds be too high and thus predicted yield spreads will be underestimated. This accounts for the EHH findings for coupon bearing bonds for recovery rate equal one. Suupose recovery rate (w) not equals to one, after doing some calculations, we can simplify the extended Merton model, proposed by EHH(2004), as

$$\begin{aligned} & BP_c(0,T) \\ = & \sum_{i=1}^{N-1} D(0,t_i)\left[\frac{Xc}{2} - f(V_T, \frac{Xc}{2})\right] \\ + & D(0,T)\left\{X(1+\frac{c}{2}) - f\left[V_T, X(1+\frac{c}{2})\right]\right\}, \quad (5) \\ & where \\ & f(V,y) \\ = & (1-w)yE^Q\left[I(V_T<K)\right] \\ + & E^Q\left[\max(y-V_T)I(V_T<K)\right] \end{aligned}$$

also, we see that, when V decreases, both terms in $f(V,y)$ increase, implying that a corporate bond can be thought of a riskless bond minus a decreasing function of V. Hence, if we use the overstated market value of assets, the corresponding corporate bond price will be overestimated.

3 Simulation

Since we have seen that the proxy estimation of market values of assets generates errors, we use another method to obtain market values of assets from market data. We follow the idea of Duan (1994) [8] of using maximum likelihood estimator (MLE) approach to estimate. We first observe a series of market values of equity (stock prices), then regard them as standard call options on market values of assets with strike price equal to its liabilities. After using MLE to find out an estimate of volatility of market value of assets, we can compute the market values of assets inversely. Using these estimates, we get predicted prices and yields of corporate bonds. By comparing the actual prices and the predicted prices, we justify the MLE approach is a good method to estimate assets volatility and market values of assets.

3.1 The MLE approach

Before going into details of our simulation procedures and results, we revisit the MLE approach proposed by Duan (1994). In fact, Duan, Gauthier, Simonato and Zaanoun (2003) [9] extended the framework to have surviviorship consideration. This feature is suitable for models with early default in each refinancing points, which is not applicable for the Merton model. Furthermore, our purpose is to compare the performances of using proxy and MLE approach, we agree that it may be better if we consider survivorship. However, for the sake of brevity, we do not consider it.

As usual, we assume that the underlying asset price evolves as the Black-Scholes dynamics, i.e.,

$$d\ln V_t = (\mu - \sigma^2/2)dt + \sigma dZ_t, \qquad (6)$$

where V_t is the market value of assets at time t, μ is the drift of the business, σ is the asset volatility and Z_t is a standard Wiener process. Under the physical probability measure, the density function of $\ln V_t$ is given by

$$\begin{aligned} & g(v_i|v_{i-1}) \\ = & \frac{1}{\sigma\sqrt{2\pi(t_i-t_{i-1})}} \\ \times & \exp\left\{-\frac{[v_i - v_{i-1} - (\mu-\sigma^2/2)(t_i-t_{i-1})]^2}{2\sigma^2(t_i-t_{i-1})}\right\}, \end{aligned} \qquad (7)$$

If we view the market value of equity (V_E) as a standard call option on market value of assets (V), then we have the risk-neutral price of the equity as

$$V_E = V\cdot N(d_1) - Xe^{-rT}\cdot N(d_2), \qquad (8)$$

where X is the book value of corporate liabilities, r is the risk-free rate, T is maturity, σ is asset volatility, $N(\cdot)$ is the cumulative distribution for a standard normal random variable and

$$d_1 = \frac{\ln(V/X) + (r+\sigma^2/2)}{\sigma\sqrt{T}},$$
$$d_2 = \frac{\ln(V/X) + (r-\sigma^2/2)}{\sigma\sqrt{T}}.$$

Since we make our inference based on the observed market values of equity, we can set the log likelihood function of μ and σ by

$$L(\mu,\sigma) = \sum_{i=2}^{n} \ln f(V_E^i|V_E^{i-1},\mu,\sigma), \qquad V_E^i \equiv V_E(t_i).$$

After applying standard change of variable technique, we obtain

$$\begin{aligned} f(V_E^i|V_E^{i-1},\mu,\sigma) &= g(v_i|v_{i-1},\mu,\sigma) \\ &\times (v_{i-1}\cdot N(d_1)|_{V=v_{i-1}})^{-1}. \end{aligned}$$

Hence, the log-likelihood function reads

$$L(\mu,\sigma) = \sum_{i=2}^{n} [\ln g(v_i|v_{i-1}) - \ln(v_{i-1}\cdot N(d_1)|_{V=v_{i-1}})]. \qquad (9)$$

We estimate parameters by solving the following optimization problem:

$$\begin{aligned} & \max_{\mu,\sigma} L(\mu,\sigma) \\ s.t. \quad & V_E(t_i) = C(t_i, V(t_i), \sigma),\ \forall\, i = 1,2,\cdots,n. \end{aligned}$$

After that, the firm values ($V(t_i)$) are obtained inversely through the constrain, one line ahead.

3.2 Simulation procedures

In this simulation exercise, we use r = 6.5%, σ = 0.25 and initial firm value of 1. One year (260-days) sample paths are generated according to the Black-Scholes dynamics of (5). We consider 4 values of debt maturities. They are 2, 5, 10 and 20 years. The face value of the debt takes three values, 0.3, 0.5 and 0.7, representing different leverage level (or creditworthiness) of a company. The higher the face value the lower the credit quality of the firm. Based on the Merton model, we compute market values of corporate equities by the standard call option formula.

Suppose the extended Merton model is a correct framework to price corporate bonds in a virtual economy. This simulation , on one hand, attempts to show that the proxy would lead to a wrong message that the extended Merton model overprices corporate bonds. On the other hand, we would like to check the performance of MLE. Therefore, we compute extended Merton prices of bonds with the generated firm values. These corporate bond values are regarded as market observable bond prices.

The remaining procedures are summarized as follows.

1. Use the market value of equity and the method of MLE to back out the asset volatility and the implied market value of assets, then plug them into the extended Merton model to find out the predicted price and credit yield spread.

2. Use the proxy to find an estimated market value of assets, and the relationship $\sigma_e = \sigma_v \frac{V_t}{S_t} \frac{\partial S_t}{\partial V_t}$ [2] to find out the 150-day historical volatility, and then use these estimates to find out the price and credit yield spread. This step follows the approach of EHH exactly.

We then compare the credit yield spreads and bond prices obtained from the two approaches.

3.3 Simulation results

The results of simulation are given in Table 1, we present the results as the percentage errors in prices, yields and credit yield spreads (comparing with true prices, yields and spreads) by using the MLE and the proxy.

Table 1 shows that average percentage errors in prices and yields of MLE approach are very close to zero, regardless of coupon rate, maturity and leverage level. This indicates that the MLE approach does a good job in estimating firm values and volatilities.

However, if we use the proxy to estimate market value of assets and asset volatility, average percentage errors in prices are positive, while those in yields and yields spreads are negative in all cases. To illustrate ideas graphically, we construct Figure 1a-d. Figure 1a plots the percentage errors of yields against debt maturities for all bonds. We see that the error points of MLE locate around zero; whereas, those of the proxy approach are always negative. Therefore, the proxy approach underestimates the yield quite substantially. We further disaggregate bonds into different credit categories in Figure 1b-d. It is seen that the proxy pricing errors are the most severe for short maturity junk bonds. This observation explains the findings of EHH.

Results obtained from our simulation further assure that the proxy approach to estimate market value of assets is inappropriate. Even though we have a correct corporate bond pricing model in hand, we still overprice corporate bonds if the proxy is used. This hidden bias against structural models in security valuation have been mentioned in Wong and Choi (2004) [10] . However, we are the first one observing its effect in corporate bond pricing models.

4 Empirical test

Evidence supporting our claim is in theoretical basis and simulation basis so far. To gain empirical evidence, this section carries out an empirical study to see if extended Merton model always overprices corporate bonds. A comparison between the proxy and the MLE approaches are made based on this set of empirical data.

4.1 Criteria of choosing bonds

Following the idea of EHH, we choose those bonds with simple capital structure and sufficient equity data. Bond prices on the last trading day of each December for the period 1986-1996 can be obtained in the Fixed Income Database. We choose non-callable and non-putable bonds issued by industrial and transportation firms. Furthermore, we exclude bonds with matrix prices and those with maturities less than one year. There are nearly 7,000 bonds that meet these criteria.

In order to have simple capital structure, we choose firms with only one or two public bonds. Also, we exclude sinkable and subordinated bonds. Next, we use a Moody's product, Rating Interactive, to further study characteristics of the firms. We choose firms with organization type as corporation and exclude those firms with non-US domicile. Furthermore, we exclude those firms with broad industry as finance, real estate finance, public utility, insurance and banking. We have 2,033 bonds on hand.

Lastly, we use Datastream to download the market value of equity from 1986 and 1996. Also, we use CompuStat to download total liabilities and equity yields throughout this period. At last, our sample contains 1,216 bonds issued by 253 firms.

4.2 Other parameters

The prices of 1,216 bonds are downloaded from Fixed Income Database. Market values of equities and dividend yields are obtained from Datastream and CompuStat, respectively. We follow the idea of EHH, to set the default point as the total liabilities of the firms, which can be obtained from CompuStat.

[2] S_t and V_t denote the market value of equity and assets at time t respectively, while σ_e and σ_v denote volatility of assets and equity at time t respectively.

For interest rate, we download the data from the Constant Maturity Treasury series reported in the Federal Reserve Board's H15 release. Since the data is available only for finite numbers of maturities, we follow EHH to use the reported interest rates to fit Nelson-Siegel (1987) [11] yield curve model. In this way, we estimate all parameters except market value of firm's assets, asset volatility, asset payout ratio and recovery rate.

For asset payout ratio (δ), we estimated it as equity payout ratio, multiplied by market value of equity (V_E) and then divided it by market value of firm's assets.(V), while equity payout ratio is obtained by annual dividend (D), and then divided it by the sum of annual dividend and market value of equity, i.e.,

$$\delta = \frac{D}{D + V_E} \cdot \frac{V_E}{V} \tag{10}$$

For recovery rate, we found that if we set recovery rate as one, we indeed ignored the role of total liabilities when pricing a corporate bond by extended Merton model. Hence, by making use of a research on recovery rate by Keenan, Shtofrin and Sobehart (1999) [12], we set the recovery rate as 51.31% in order to respect the role of total liabilities. EHH also employed this approximation.

Lastly, for the market value of firm's assets and asset volatilities, we use two approaches to estimate. Under the proxy approach, market value of firm's assets are equal to sum of market value of equity and total liabilities, and assets volatilities are estimated using 150 historical volatility that has been discussed above. Under the MLE approach, for each firm and each year, we use one-year series of market value of equity and the average value of riskfree rate throughout that year to estimate the asset volatility, and then use this value to back out market value of assets.

4.3 Empirical results

The empirical results are summarized in Table 2, Table 3 and Table 4. We observe that the average percentage errors in pricing are 2.38%, by the MLE, and 7.66%, by the proxy approach. For the average percentage errors in yields, the percentage errors are -1.01% and -15.26% with MLE and the proxy respectively. For the average differences between predicted yields and true yields, the errors are 3 and -130 basis points by the MLE and proxy approach respectively. The results show that the average percentage errors in pricing and yields by the MLE approach are not as severe as those by the proxy. However, the MLE approach gives very accurate estimations of yields on average.

For absolute pricing errors, we can see that there are 57 and 17 firms with errors less than 1%, 571 and 446 firms less than 5%, and 935 and 890 firms less than 10% respectively by using MLE and proxy approach respectively. For absolute percentage errors in yields, we can see that there are 20 and 4 firms with errors less than 1%, 146 and 33 firms less than 5%, and 449 and 250 firms less than 10% respectively by using MLE and proxy approach respectively. For differences between predicted yields and true yields, there are 25 and 5 firms with errors less than 10 basis points, 215 and 74 firms less than 50 basis points and 647 and 477 firms less than 100 basis points. We can see that the numbers of firms within a certain errors is greater by using the MLE approach than those by proxy. Hence, these results further justify that our MLE approach to estimate market value of assets and asset volatilities is better than traditional proxy approach.

5 Conclusion

This article shows that the importance of choosing a suitable method to estimate market value of firm's assets and asset volatilities when using structural models to price corporate bonds. By using theoretical arguments and implementing simulation, we show that the use of proxy approach to estimate market values of assets implicitly overestimate corporate bonds. An MLE approach is proposed to estimate market values of assets when testing validity of structural models. Lastly, our empirical results suggest that the Merton model may work well not only if we should find out accurate estimates of assets payout ratios and recovery rate, but also use the MLE approach to estimate market values of assets and asset volatilities.

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Table 1:
Simulation results of percentage errors in prices, yields and spreads by using MLE and proxy to estimate market value of assets

	by MLE approach:			by proxy:		
	% error in prices	% error in yields	% error in spreads	% error in prices	% error in yields	% error in spreads
Characteristics	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)
Panel A: Different levels of coupon rate:						
c = 0%	0.06%	-0.10%	-4.14%	1.37%	-2.53%	-93.22%
	(0.10%)	(0.18%)	(3.08%)	(2.04%)	(5.48%)	(12.81%)
c = 8%	0.03%	-0.09%	-4.09%	0.75%	-2.35%	-92.28%
	(0.05%)	(0.18%)	(3.96%)	(1.26%)	(5.64%)	(14.64%)
Panel B: Different levels of total liabilities:						
X = 0.3	0.00%	-0.01%	-2.92%	0.15%	-0.22%	-86.89%
	(0.01%)	(0.01%)	(3.84%)	(0.31%)	(0.48%)	(20.98%)
X = 0.5	0.03%	-0.05%	-4.08%	0.77%	-1.64%	-94.68%
	(0.04%)	(0.07%)	(2.56%)	(1.05%)	(3.08%)	(7.12%)
X = 0.7	0.10%	-0.22%	-5.35%	2.25%	-5.45%	-96.69%
	(0.11%)	(0.26%)	(3.69%)	(2.33%)	(8.28%)	(4.88%)
Panel C: Different levels of time to maturities:						
T = 2	0.03%	-0.15%	-4.50%	0.70%	-4.21%	-93.77%
	(0.05%)	(0.28%)	(5.08%)	(1.44%)	(8.95%)	(22.86%)
T = 5	0.05%	-0.11%	-3.86%	1.18%	-3.06%	-95.34%
	(0.07%)	(0.18%)	(2.66%)	(1.86%)	(5.36%)	(5.70%)
T = 10	0.05%	-0.07%	-3.83%	1.26%	-1.71%	-91.83%
	(0.09%)	(0.11%)	(2.71%)	(1.84%)	(2.69%)	(8.66%)
T = 20	0.06%	-0.04%	-4.27%	1.10%	-0.77%	-90.08%
	(0.10%)	(0.06%)	(3.15%)	(1.69%)	(1.12%)	(10.67%)

Table 1 presents the mean and standard deviation (in percentages) of percentage errors in prices, yield and credit yield spreads using the MLE approach and proxy when using simulation data. The percentage errors in prices are calculated as predicted prices minus true prices, and then divided by true prices, similar calculations for percentage errors in yields and spreads

Table 2

Empirical results of percentage errors in prices and yields, difference between predicted yield and true yield by using MLE and proxy to estimate market value of assets

Methods of estimation	Mean of % errors in prices (Standard deviation)	Mean of % errors in yields (Standard deviation)	Mean of difference between predicted yield and true yield (Standard deviation)
proxy	7.66% (6.95%)	-15.26% (13.77%)	-1.30% (1.28%)
MLE	2.38% (9.78%)	-1.01% (36.18%)	0.03% (3.36%)

Table 2 presents the mean and standard deviation (in percentages) of the percentage errors in prices and yield using the MLE approach and proxy when using empirical data. The definition of percentage errors in prices and yields are the same as those in table 1

Table 3

A comparison of number of firms with percentage errors in prices and yields within a certain range

Method of estimation	Number of firms with errors within 1%	Number of firms with errors within 5%	Number of firms with errors within 10%
Panel A: Percentage errors in prices:			
proxy	17	446	890
MLE	57	571	935
Panel B: Percentage errors in yields:			
proxy	4	33	250
MLE	20	146	449

Table 3 presents the number of firms with percentage errors in prices and yields within one, five and ten percentage when using MLE and proxy to estimate market value of assets. The definition of percentage errors in prices and yields are the same as those in table 1.

Table 4

A comparison of number of firms with differences between predicted yields and true yields within a certain range

Method of estimation	Number of firms with errors within 10 bps	Number of firms with errors within 50 bps	Number of firms with errors within 100 bps
Panel A: Percentage errors in prices:			
proxy	5	74	477
MLE	25	215	647

Table 4 presents the number of firms with differences between predicted yields and true yields within ten, fifty and one hundred basis points (bps) when using MLE and proxy to estimate market value of assets. The definition of differences between predicted yields and true yields are predictes yields minus true yields.

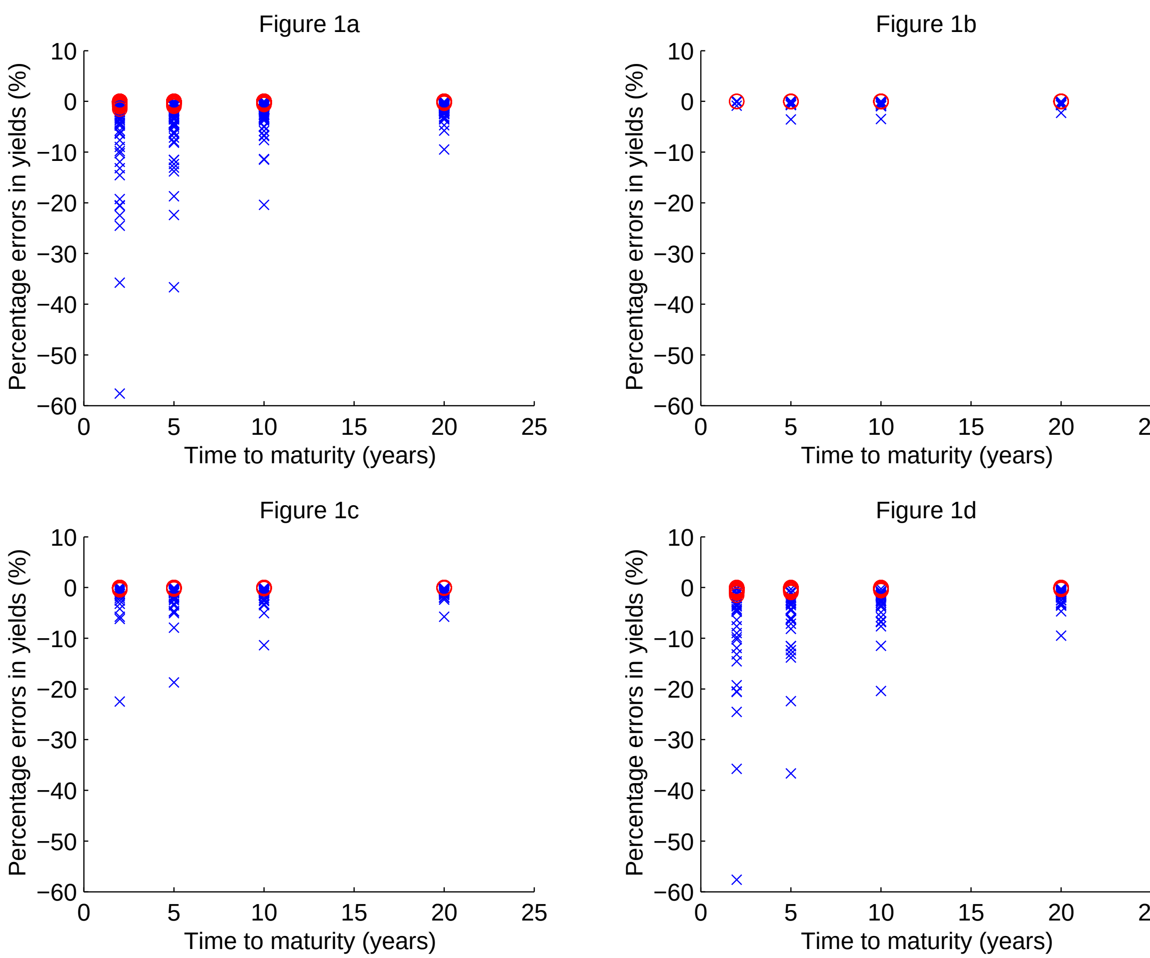

Figure 1. This figure shows the percentage errors in yields when using MLE and proxy to estimate market values of assets. In all figures, ‘o’ indicates percentage error in yield when using MLE, while ‘x’ indicates percentage error in yield when using proxy. Figure 1a plots the percentage errors of yields against debt maturities for all bonds. Figure 1b, 1c and 1d plot those bonds with high, medium and low credit categories respectively.