Turing Trilogy 圖靈三部曲
Part 1

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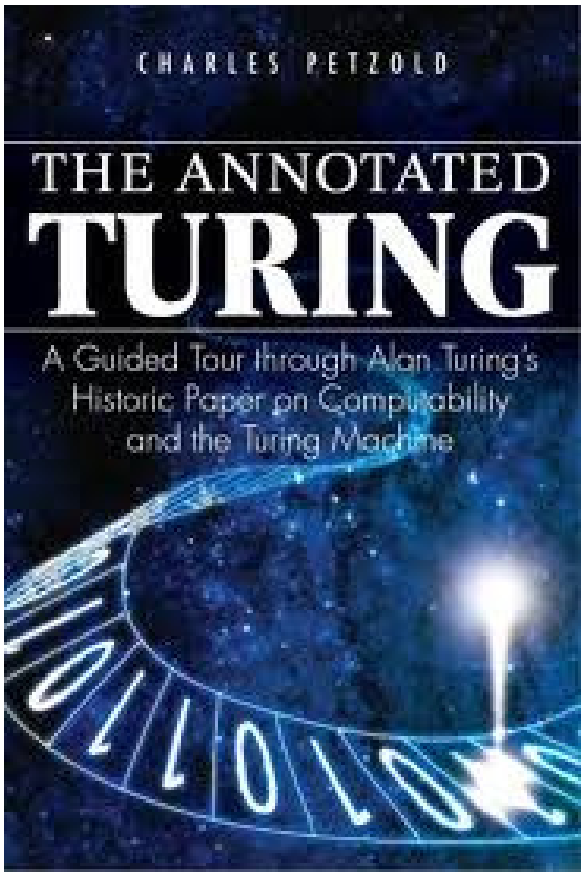
And Hong Kong Computer Society

One of them is the speaker

Some observations

- Few people know about Alan Turing, including IT professionals
- Few people read original scientific/mathematical papers
- Never was so much owed by so many to **Alan Turing (my view)**

The Annotated Turing



Charles Petzold, (1953 –)
Microsoft MVP

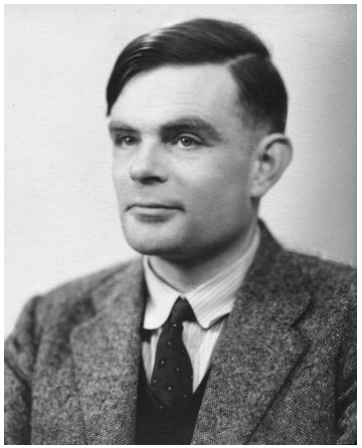
The Book

- Part I – Background and Foundation
- Part 2 – The tool for the proof (Turing Machine)
- Part 3 – The proof on Decidability
- Part 4 – And Beyond

- Arthur's objective: Makes that paper accessible to a much broader audience (other than computer science majors, programmers, and other techies)
- Part 3 is very difficult. Reader may consider skipping it.

Alan Turing (1912-1954)

- His life and works
- Background and foundation
- Detail look at his major contribution
- Personal remarks on Alan Turing
- ATY2012

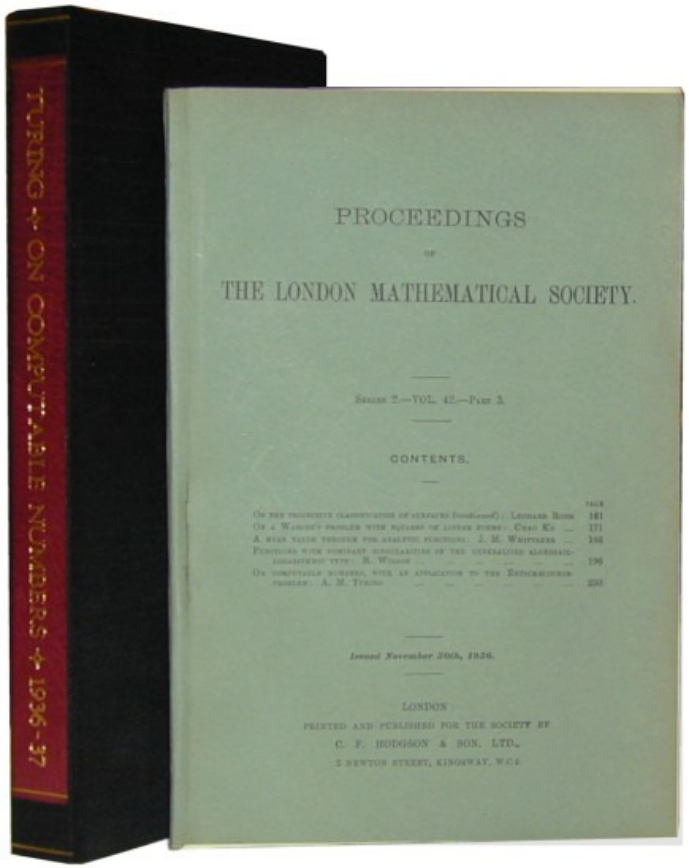


The brief history of Alan Turing

- 23 June 1912 – Born in London. Father is a civil servant in India. Brought up in UK with a retired Army couple
- 1926-1931 – Study at Sherborne Boys
- 1931-1936 – King's College, Cambridge
- 1936-1938 – PhD at IAS, Princeton
- 1939-1945 – Cryptanalysis at GCCS, Bletchley Park
- 1945-1947 – Working on ACE at NPL, London
- 1948-1952 – Working on Mark 1, Manchester U
- 1952-1954 – Working on Mathematical Biology, Manchester U and conviction of gross indecency
- 7 June 1954 – Commit suicide with a cyanide poisoning apple

His Works and Contributions

- 3 Major Contributions
  - Universal Turing Machine, lay the theoretical foundation of modern computer
  - Breaking the Enigma code, shorten the war by 2 years and saving millions of life
  - Turing Test, in search of artificial intelligence



On Computable Number,

with an Application to the Entscheidungsproblem

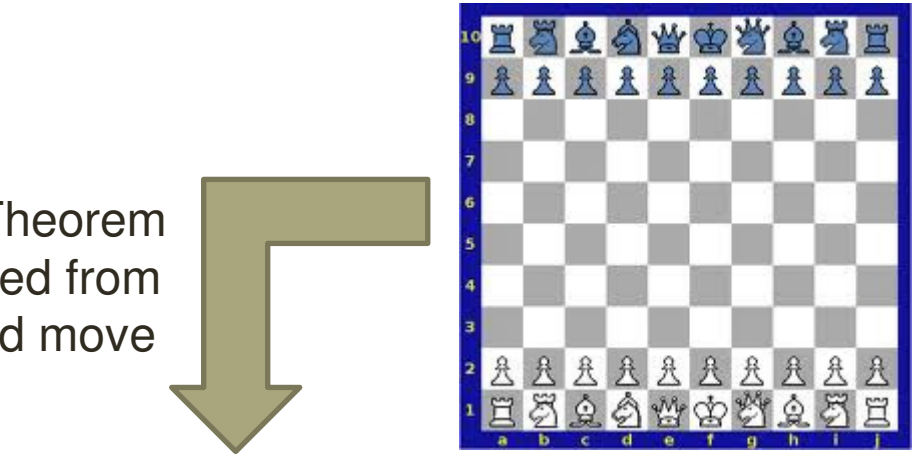
12 Nov 1936, Proceedings of the London Mathematical Society

The Background

- By end of 19th century, an optimism that scientists were on the verge of total knowledge except a few remaining riddles
- David Hilbert 23 outstanding math problems
  - No. 1: Cantor's continuum hypothesis
  - No. 2: The compatibility of the arithmetical axioms, the completeness of consistency of formal system
  - No. 10: the decidability of general process in solving Diophantine (eg. Fermat Last Theorem, Goldbach's Conjecture etc.)
- 3 Approaches to Math Foundation Crisis
  - Russell/Whitehead – Logicism (Principia Mathematica)
  - L. E. J. Brouwer - Intuitionism
  - David Hilbert (Hilbert's Program) – Formal System to lay a solid foundation on mathematic in late 20s

An analogy of Formal System

Axiom 公理: Initial Configuration



Not Theorem
Derived from
Invalid move

非定理

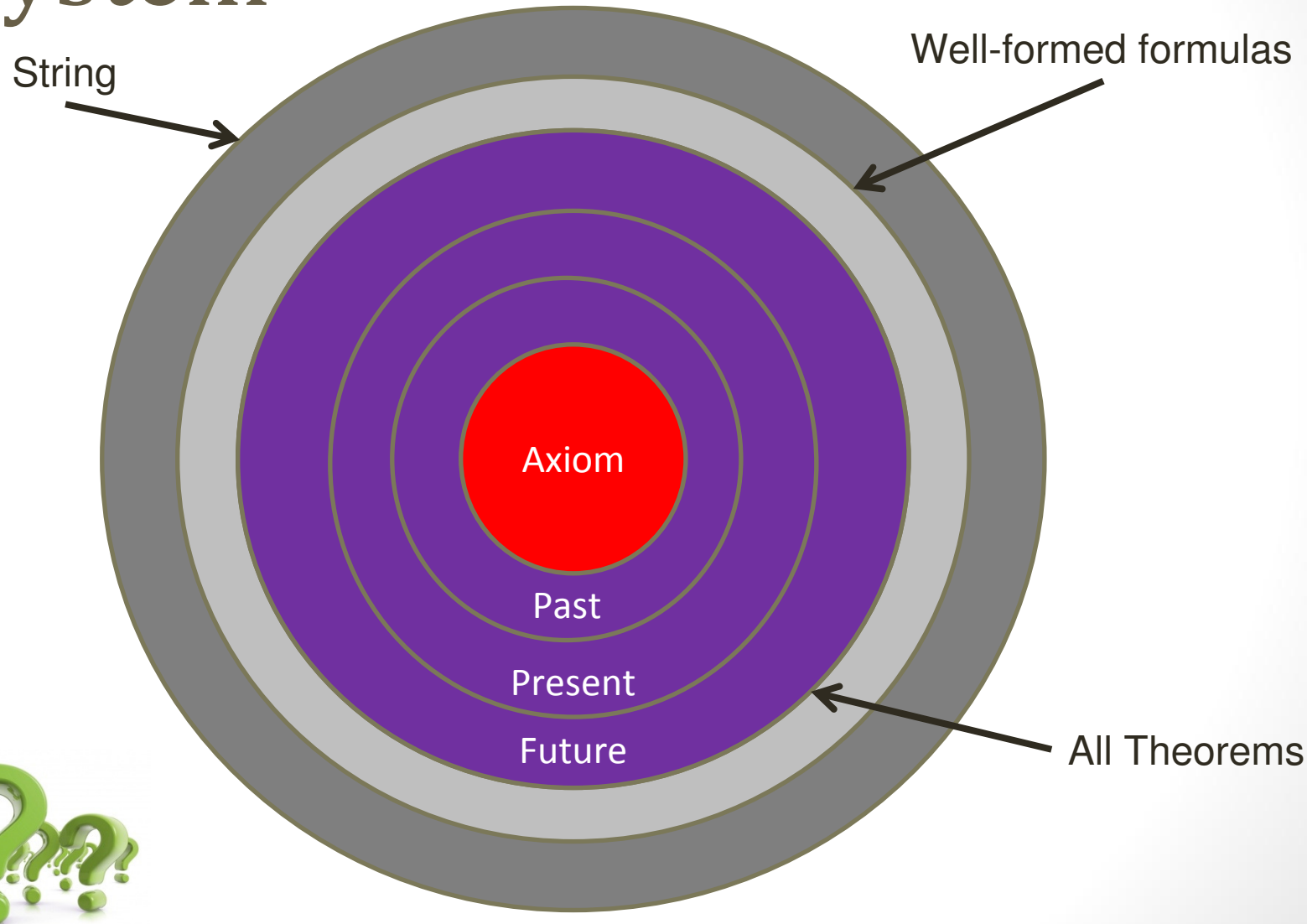
Theorem
Derived from
Valid move

定理

Formalism expected qualities

- Independence, no axiom can be derived from other axioms
- Consistency, the impossibility to derive two theorems that contradict each other
- Completeness, the ability to derive all true formulas from the axioms
- Decidability, a general method to detemine the provability of any given Well-Formed Form

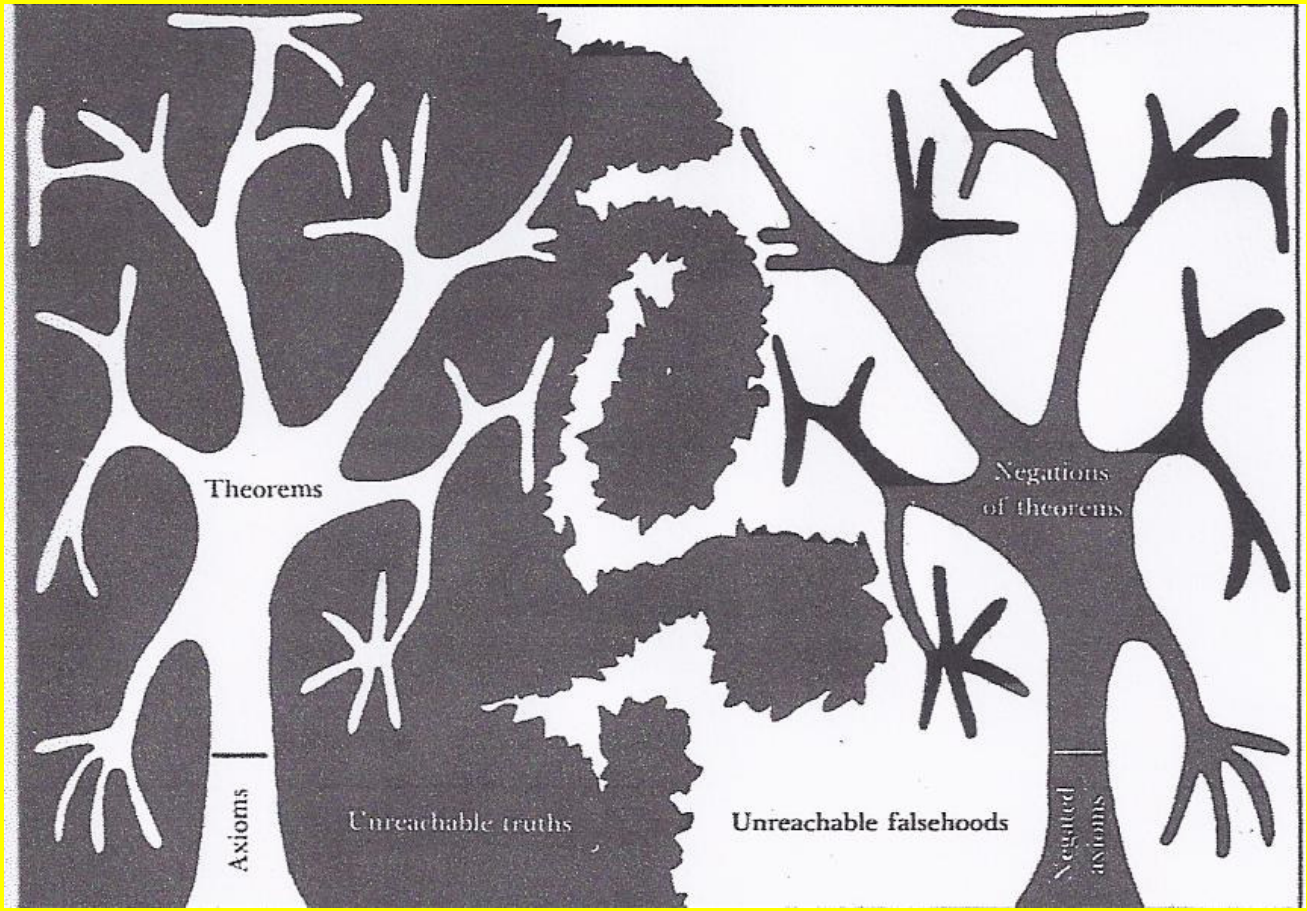
Hilbert Dream on Formal System



1905-1936: All things collapse

- 1901: Russell, Russell's Paradox
- 1905: Albert Einstein, Theory of Relativity
- 1925: Quantum Mechanics (everything is probability): Planck 普朗克, Heisenberg , Schrödinger 薛丁格, Dirac 狄拉克
- 1927: Heisenberg Uncertainty Principle 海森堡不確定性原理
- 1931: Godel **哥德爾**, Incompleteness Theorem
- 1936: Turing, Undecidability on first order logic and lay the theoretical foundation of modern computer science **as a by product**

Godel Incompleteness Room



Well-formed formulas

Strings

Unprovable Theorem?

- Fermat's Last Theorem 費瑪最後定理
  - $x^n + y^n = z^n$
  - No integer solution for n > or = 3
  - 300 years to prove
  - Andrew Wiles (Shaw Prize Laureate 2005)

- Goldbach's Conjecture 哥德巴赫猜想
  - Every even integer greater than 2 can be expressed as the sum of two primes: 1+1
  - Chen Jingrun (陳景潤): 1+2, sufficiently large

Real Numbers

Real Numbers 實數

Rational Numbers 有理數

Irrational Numbers 無理數

Integers 整數 (eg. -3, 0, 7)	Faction 分數 (eg. 1/3, 7/2)	Irrational Algebraic 無理代數數 (eg. $\sqrt{2}$, $\sqrt{3}$)	Transcendental 超越數 (eg. e, Π)

Algebraic Numbers代數數

Computable Numbers 可計算數

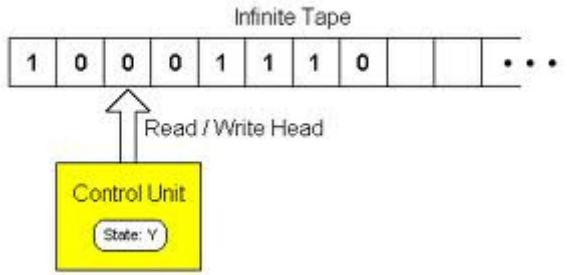
Definable Numbers可定義數 (eg. Halting Machine DN, Chaitin Ω)

What is the goal of 1936 paper?

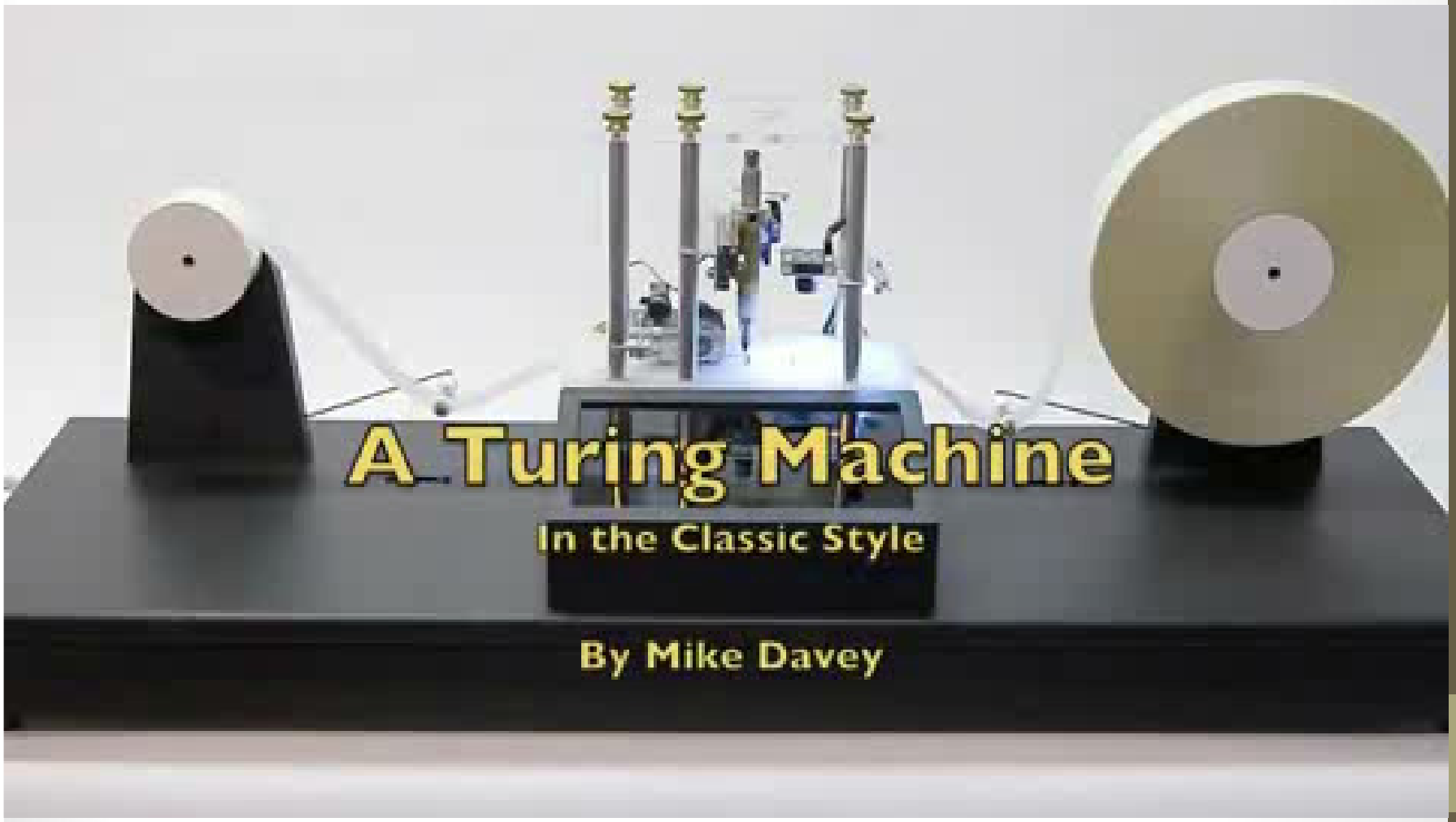
- To prove the undecidability of first order logic, which is complete if it is logically valid (proven by Godel in 1929)
- A non-conventional approach:
  - Build a mathematical tool, an imaginary computing device (later called Turing Machine)
  - Using the tool to proof the undecidability
- The proof is useful but the tool is more interesting and fruitful
- Note 1: the undecidability is proven by Alonso Church earlier
- Note 2: this is not the proof of Hilbert 10th which is solved in the 70s by a Russian

Turing Machine (TM) 圖靈機

- An infinite tape
- A tape read/write head
- A finite action table
  - Start state
  - Symbol
  - Operations (write, head movement)
  - New state
- A state register (with a start state)



Turing Machine in Action



TM without input parameter

- One TM computes one computable number
- Eg. 1/3 = 0.01010101……. (binary number)
- A TM that computes the digits after the decimal of 1/3 in binary number, starting with a blank tape

Configuration		Behaviour		
Initial State	Head Read	Head Write	Head Move	New State
S1	Blank	0	R	S2
S2	Blank	1	R	S1

- This will continue forever
- This is a circle free TM

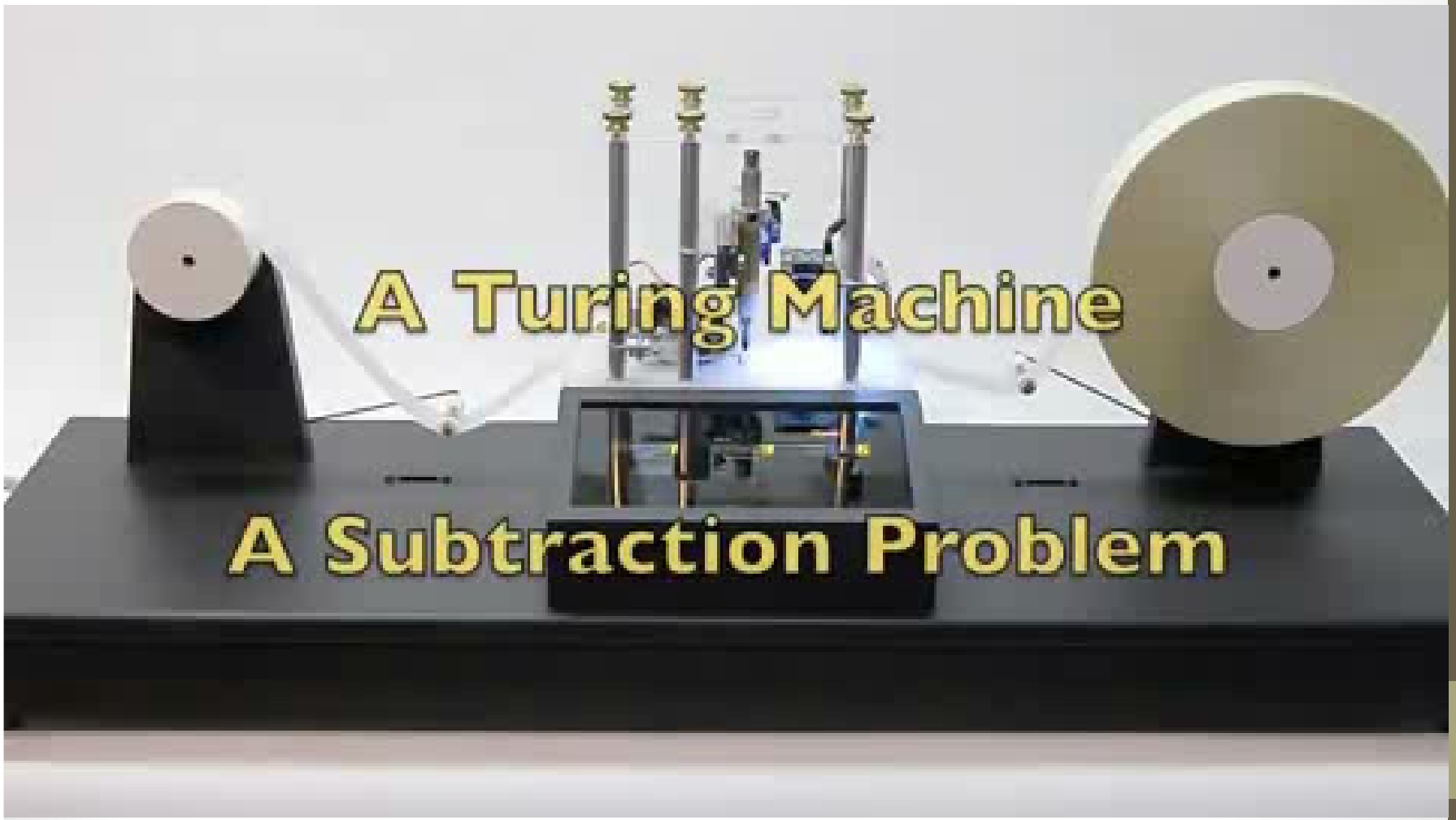
- I call this TM-A

TM with input parameters

- Another Turing Machine: TM-B
- One TM do arithmetic operation on unary numbers
- Unary number: 0 is 1, 1 is 11, 2 is 111, 3 is 1111
- A TM that adds two unary numbers together
- eg. Add 2 and 3 (ie. 111 + 1111), 111 and 1111 are on the tape separated by 1 blank space

Configuration		Behaviour		
Initial State	Head Read	Head Write	Head Move	New State
S1	1	Blank	R	S2
S2	1	1	R	S2
S2	Blank	1	R	S3
S3	1	1	R	S3
S3	Blank	Blank	L	S4
S4	1	Blank	-	Stop

A subtraction TM (TM-C)



3 – 2 = 1 (decimal) or 111 – 11 = 1 (unary)

Transformations

- This table can be turned into a character string

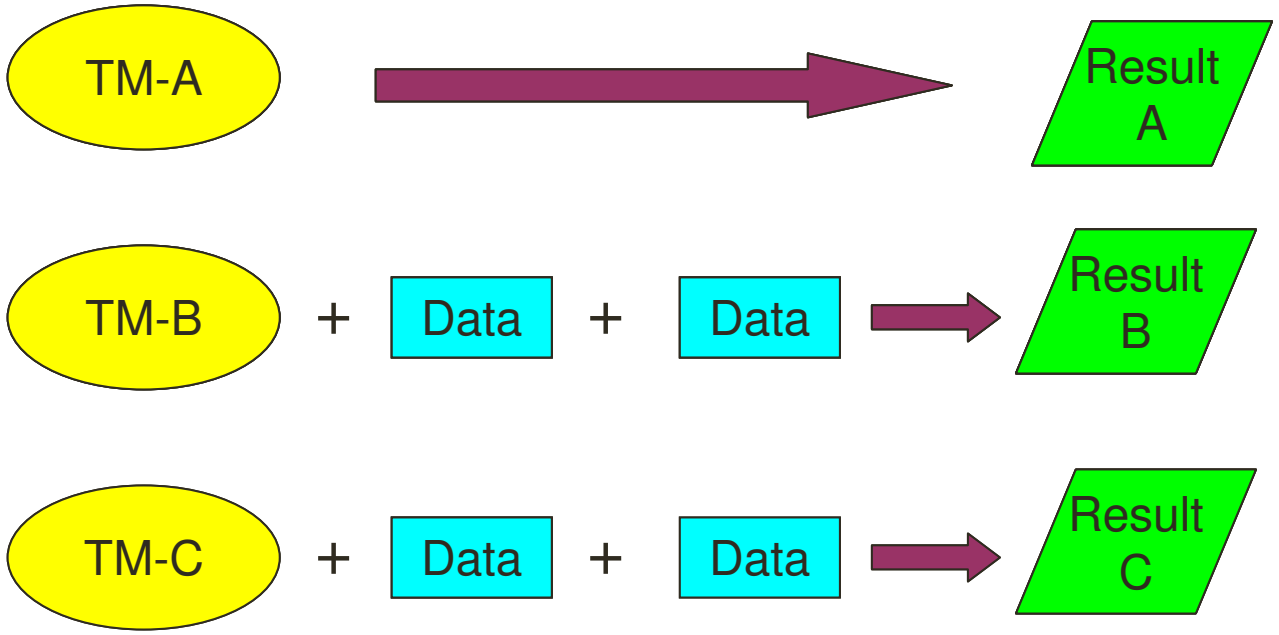
Configuration		Behaviour		
Initial State	Head Read	Head Write	Head Move	New State
S1	Blank	0	R	S2
S2	Blank	1	R	S1

- DADDCRDAA;DAADDCCRDA;;
- Now, Let D=4 A=1 C=3 R=5 L=6 ;=9
- This string can be shown as integer (Description No.)
- 41443541194114433541 99
- This is the DN of TM-A, I call it DN-A

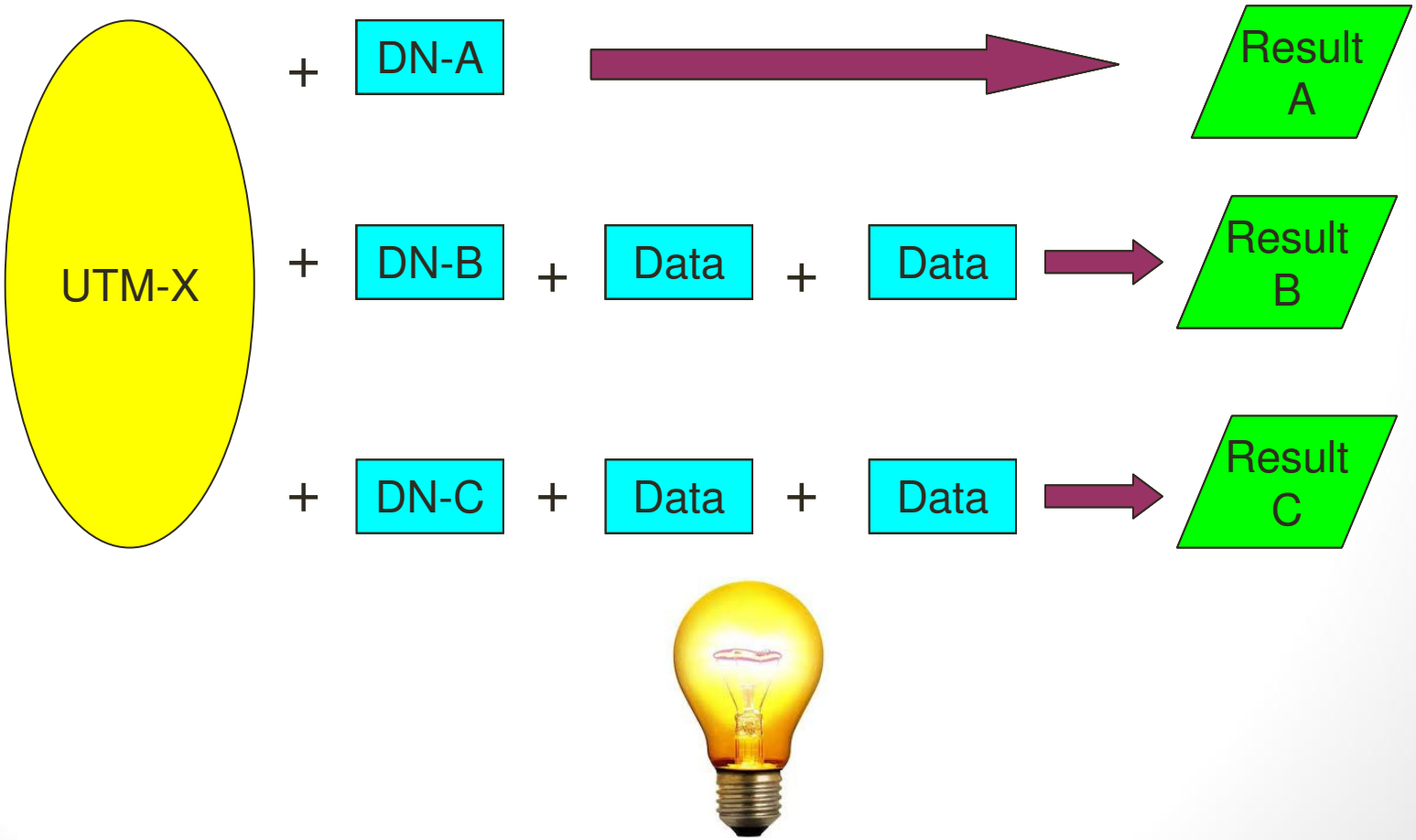
Universal Turing Machine
通用圖靈機

- A Turing Machine that can read any Description Number (DN) as input
- Simulate the TM* represented by the DN
- Generate the output as it is generated by TM*
- The theoretical framework of modern computer:
  - Program stored as data
  - Program as input to another program (multi-levels)
  - General Purpose Computer

TM is a special purpose machine



UTM is a general purpose machine



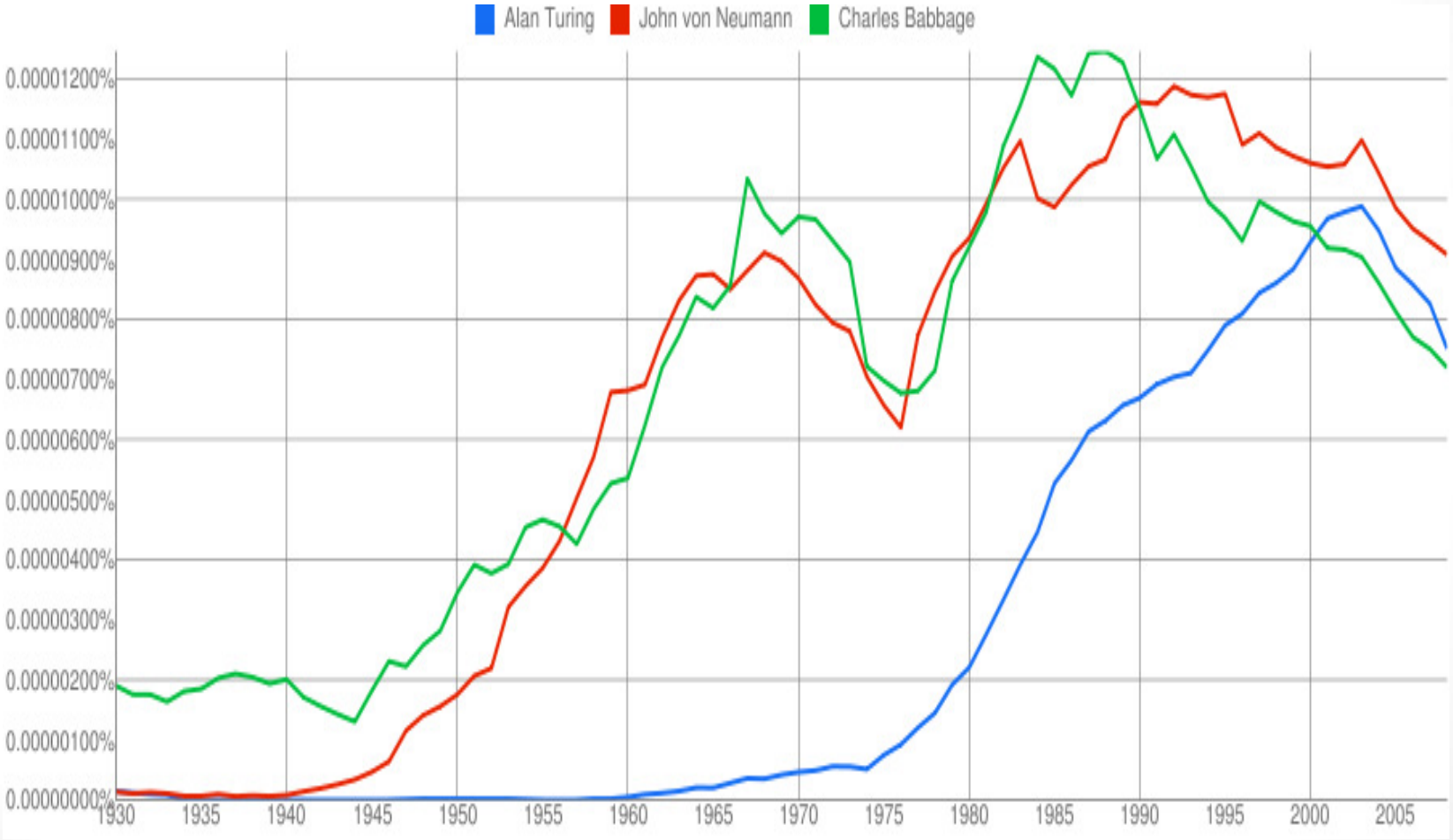
The findings and implications

- An irrational number can be represented as an integer
- An algorithm (TM) is in fact a number to another TM (UTM 1) which can be another number to another TM (UTM 2) and so on…
- A TM has its limitation: it cannot determine the fate of another TM (Undecidable)
- Computable Number is enumerable
- All algebraic numbers and some transcendental numbers are enumerable and computable
- All computable numbers are definable but not vice versa
- Not all real numbers are definable
- Computable functions (eg. SIN X) performed by TM

Limits of computer

- No computer or programming language known today is more powerful than the TM
- No computer or programming language can solve the halting problem
- No computer or programming or programming language can determine the ultimate destiny of another computer program

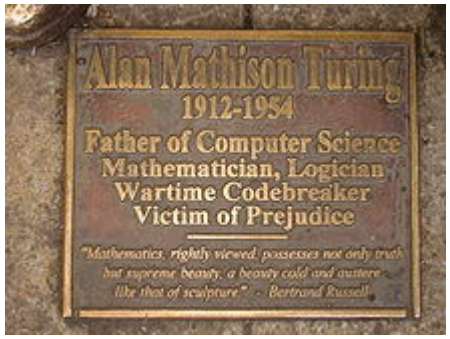
Who is the Father of Computer?



Personal remarks on Alan Turing

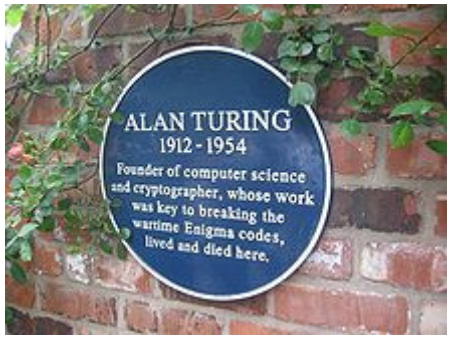
- UK vs USA on who invents the computer (Turing vs von Neumann)
- UK vs USA on the post war development strategy on technology
- National secret (cold war) and Homosexuality (lose of clearance)
- Math/Logic vs Engineering/Business Government keep his injustice story low profile
- 1936 paper is a watershed event
  - Define a new direction
  - Liberate people's creativity
  - Long lasting effect, people still working on it
- WWII effort
  - Not just shorten the war and save a lot of life
  - History may be very different if UK lost in 1941

In memory of Alan Turing



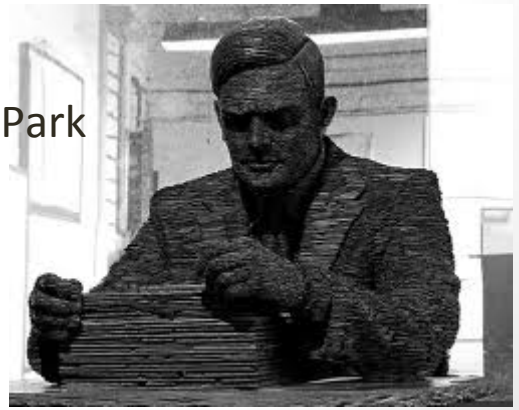
Turing memorial statue plaque in Sackville Park, Manchester

Turing's last home at Wilmslow, Cheshire



Sackville Park Manchester

Bletchley Park



Recognition and Tributes

- Establishment of Turing Award in 1966 by ACM, equivalent of Nobel Prize in Computer Science
- Second most significant alumnus in the history of Princeton University, after President James Madison
- Named one of the 100 Most Important People of the 20th century by Time Magazine, 1999
- Ranked 21st on the BBC nationwide poll of the 100 Greatest Britons, 2002
- 10 Sept 2009: Public apology by Gordon Brown
- May 2011: one of the six great UK/USA scientists mentioned in Obama historic speech in Westminster

Obama Westminster's speech

2012 Alan Turing Year

- It is time to celebrate the life and scientific influence of Alan Turing on the occasion of the centenary
- Growing list of countries planning Alan Turing Year celebrations
- What about HK?
  - Exhibition/Competition
  - Book/Article
  - Seminar/Talk
  - Play
  - Arts
  - Marathon
  - Facebook Group

Reference

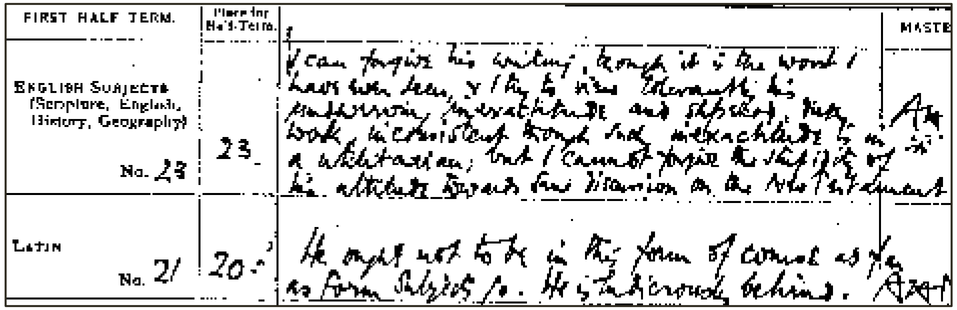
- Charles Petzold, “The Annotated Turing: A Guided Tour Through Alan Turing’s Historic Paper”
- Simon Singh, “Code Book”
- Douglas R. Hofstadter, “Godel, Escher, Bach: An Eternal Golden Braid”
- Andrew Hodges, “Alan Turing: the enigma”
- Alan Turing: http://plato.stanford.edu/entries/turing/
- Turing Test: http://plato.stanford.edu/entries/turing-test/
- Obama Speech: http://www.youtube.com/watch?v=0bac5mL3Y_o [16:28 – 17:35]

QUESTIONS?

Alan Turing (1912-1954)

1926-31: Sherborne School

Poor Results



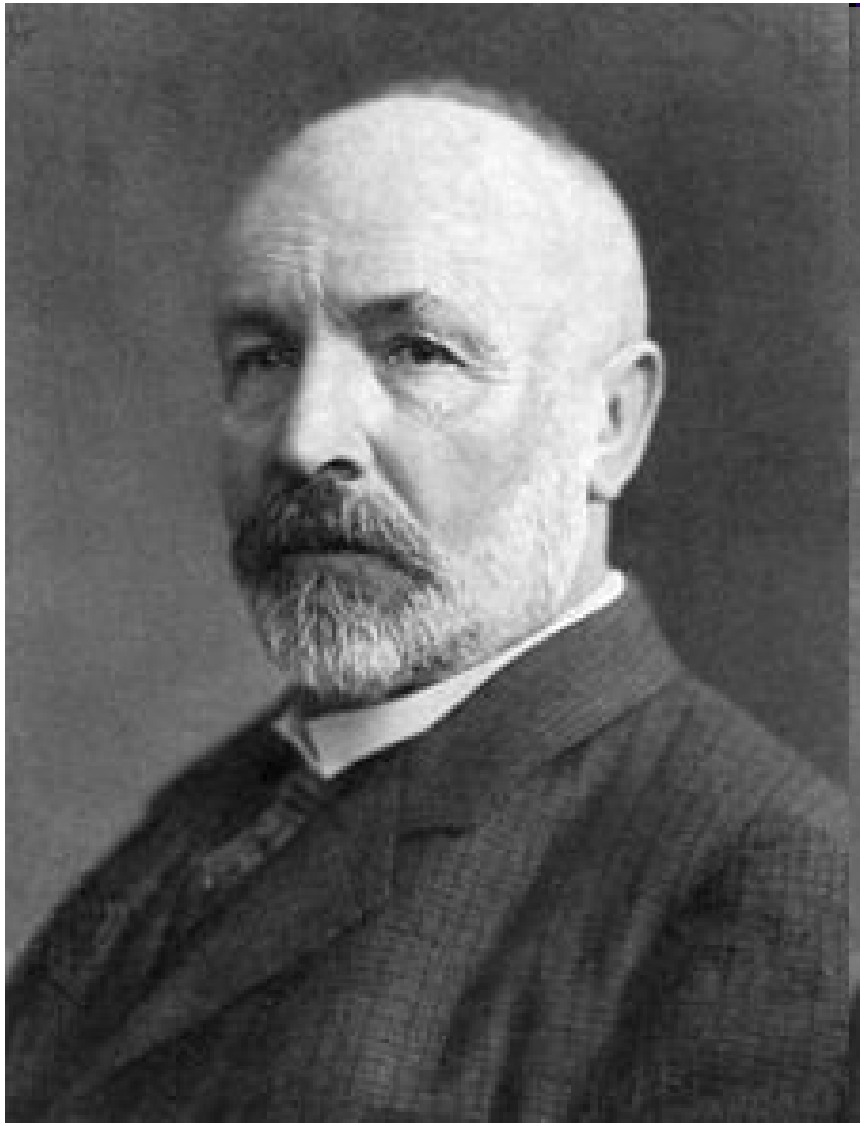
FIRST HALF TERM.	Place in Half-Term.		MASTE
ENGLISH SUBJECTS (Scripture, English, History, Geography) No. 23	23	I can forgive his writing, though it is the worst I have ever seen, [illegible]	[illegible]
LATIN No. 21	20	He ought not to be in this form [illegible]	[illegible]

- English: Bottom of the class.
- Latin : only second from bottom
- The maths and science: 'His work is dirty'.
- He was nearly stopped from taking the O-Level

His classmate Christopher Morcom (died in 1930) has great influence on him

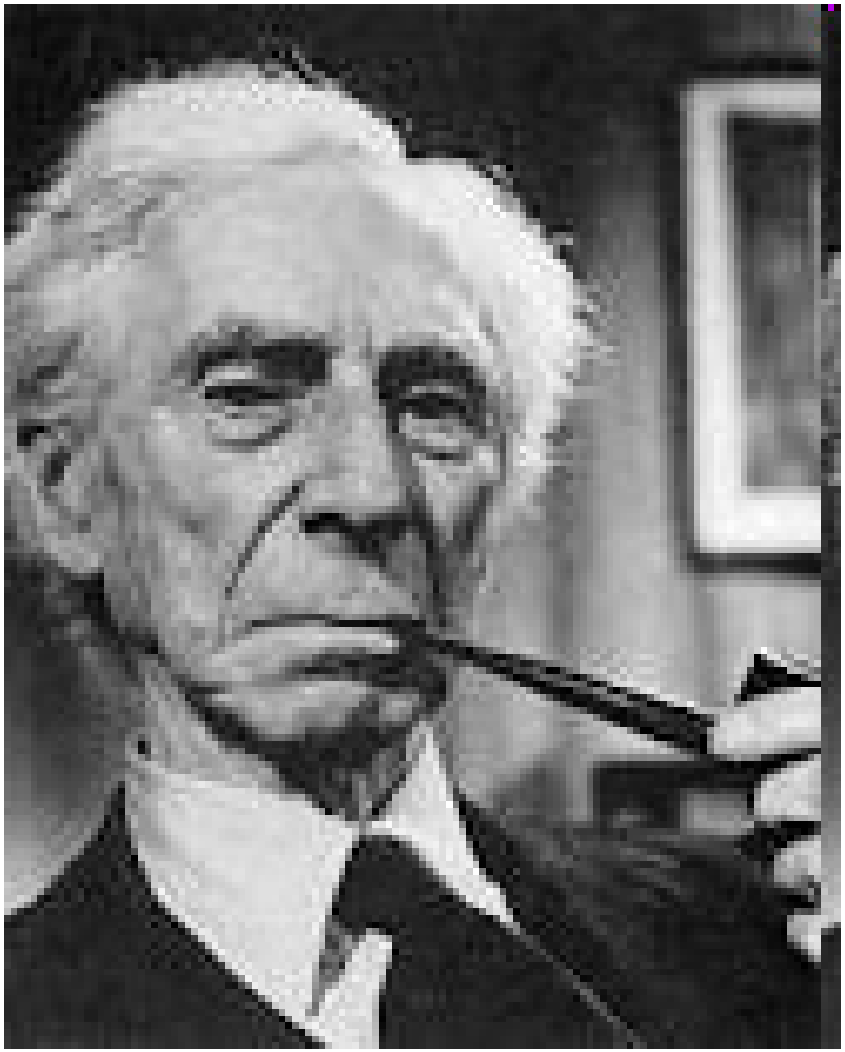
David Hilbert 希爾伯特

1862-1943



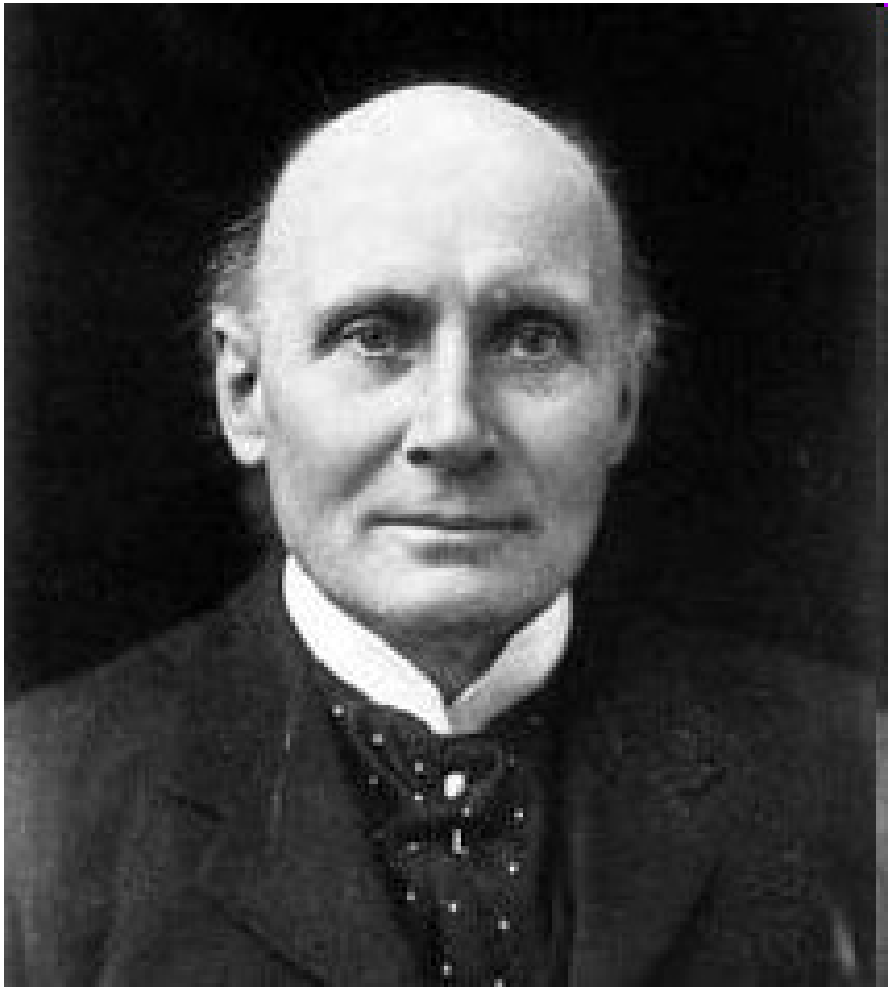
Georg Cantor 康托爾

1845-1918



Bertrand Russell 伯特蘭·羅素

1872-1970

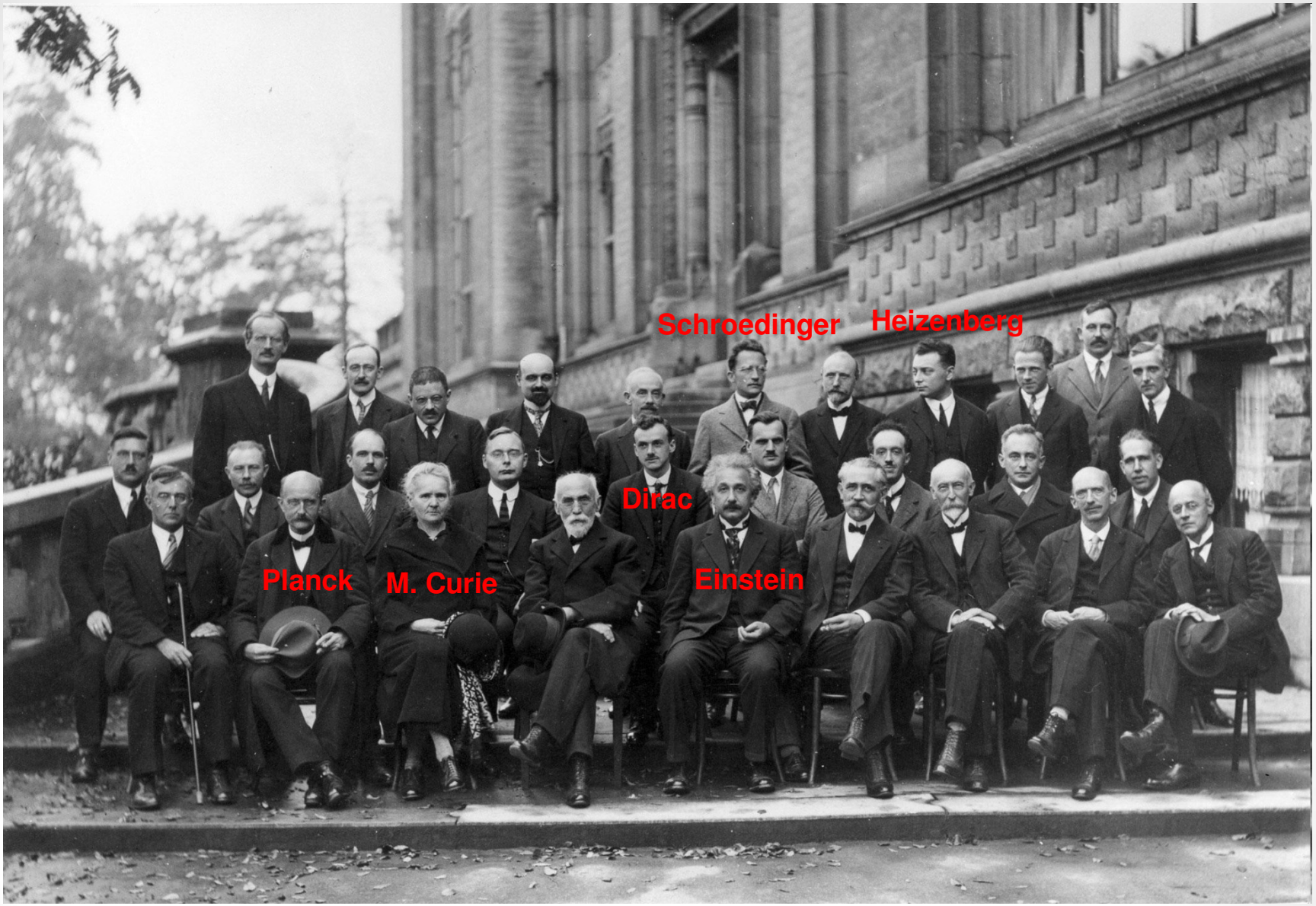


Alfred Whitehead 懷特黑德

1861-1947

L E J Brouwer 魯伊茲·布勞威爾

1881-1966



3rd (l/r): A. Piccard, E. Henroit, P. Ehrenfest, E. Herzen, Th. De Donder, E. Schroedinger, E. Verschaffelt, W. Heizenberg, R.H. Fowler, L. Brillouin

2nd (l/r): P. Debye, M. Knudsen, W.L. Bragg, H.A. Kramers, P.A.M. Dirac, A.H. Compton, L. de Broglie, M. Born

1st (l/r): I. Langmuir, M. Planck, M. Curie, H.A. Lorentz, A. Einstein, P. Langevin, Ch.E. Guye, C.T.R. Wilson, C.W. Richardson

Kurt Gödel 哥德爾

1906 - 1978

陳景潤

1933 - 1996

Alonso Church 邱奇

1903 – 1995

John von Neumann 馮·諾伊曼

1903 – 1957