

# MATH 2221 **B** Mathematics Laboratory II

## Final Examination

Name: \_\_\_\_\_

Student ID.: \_\_\_\_\_

In this test, the [color version](#) of this assignment can be obtained from the **H:** drive.

Please read the following carefully:

### General Guidelines for Test Submission.

- Write your FULL name and student ID number legibly on the cover sheet (otherwise we will not take any responsibility for your test). Please write your answers using a black or blue pen, NOT any other color or a pencil.
- Please go to the class and lab indicated by your registered course code via the CUSIS system. If you go to a different lab than the one you are registered for, you will not receive credit for the test even if you completed it.
- The use of cellular phones/graphing calculators/iPads will NOT be permitted during the test. Please do not use our lab computer to recharge your cellular phone battery. No photo taking is allowed in the test.
- If you are found cheating in the test, you will automatically get an F grade in this test and your act will be reported to the Department for necessary disciplinary actions.
- During the test, you can freely access the `help` command. All internet links for the outside world will be disconnected, including our course website. *You are allowed to bring book, lecture notes, formula sheets, etc. to the test. But they should be put on the desk before the test begins. Once the test starts, you are not allowed to put any additional things on your desk or take anything out of your bag.*
- Answer all questions. Show work to justify all answers.
- The quiz lasts 105 minutes.
- Please save all your codes in the MATLAB m-file. We will not take any responsibility for your m-files if you did not save your works. There will be no way to make up m files after the test.

- If you finish early, please remain in your seat so that you do not disturb others.
- When time is up, stop writing and pass your answer sheet to the aisle. Please stay in your seat until all answer sheets are collected.
- Please sign and date the statement of Academic Honesty.

Please read and sign the following declaration before handing in your test. Otherwise, no marks will be given.

I declare that the assignment here submitted is original except for source material explicitly acknowledged. I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained on the website <a href="http://www.cuhk.edu.hk/policy/academichonesty/">http://www.cuhk.edu.hk/policy/academichonesty/</a>	
_____ Signature	_____ Date

1 (15 marks)		4 (15 marks)	
2 (15 marks)		5 (20 marks)	
3 (15 marks)		6 (20 marks)	
		Total	/100 pts

## Exercises

**Note:** Remember to add ";" if you want to suppress the output. Marks will be deducted if there is any unnecessary output. Unless specified in the question, you should not print anything on the screen.

1. (15 marks) Define a recursive sequence  $\{x_n\}$  by

$$x_{n+1} = \frac{13}{x_n} + \frac{x_n}{24},$$

where  $x_1 = 4$ .

Write a **recursive** function `q1(N)` to calculate  $x_N$  by calling function itself. Save the function in the directory H:.

### Instructions

- (a) Input of this function is the number of iteration  $N$  and output is the value of  $x_N$ ;
- (b) Indicate the  $n$ -th iteration by showing "step n",  $n$  is shown as an integer whose total number of characters is 3. Also show the value of  $x_n$  in each iteration with 6 decimal places, where total numbers of space is 7.

**Note:** In order to show the result as Sample Output, Please call the function itself only ONE time in each iteration, i.e., try to avoid using `q1(n)` twice in generating `q1(n+1)`.

### Sample Output

```
>> b = q1(4)
step  1,  x = 4.000000
step  2,  x = 3.416667
step  3,  x = 3.947239
step  4,  x = 3.457909

b =

    3.4579
```

## Solution

```
function x = q1(n)
% q1
    if (n == 1)
        x = 4;
        fprintf('step %3i, x = %7.6f\n', n, x)
    else
        r = q1(n-1);
        x = r/24 + 13/r;
        fprintf('step %3i, x = %7.6f\n', n, x)
    end
end
```

2. (15 marks) Write a script `q2.m` to complete the following tasks.

- Define  $u = -x^2 - 5x$ ,  $v = x^2 - 3$ . where  $x$  is a symbolic variable.
- Find the  $x$ -coordinate of the intersection points of the curves  $y = -x^2 - 5x$  and  $y = x^2 - 3$ ,  $x_1, x_2$ , where  $x_1 < x_2$ .
- Find the area between the curves  $y = -x^2 - 5x$  and  $y = x^2 - 3$ , save your answer in a variable  $A$ .
- Find the equation of tangent of  $y = -x^2 - 5x$  at  $x = 2$ . Then set  $z = mx + c$ , where  $m, c$  are answers computed by you,  $x$  is a symbolic variable.  
**Hint:** The equation of tangent is  $y = mx + c$ . Try to find  $m$  and  $c$ .
- Find the volume of solid obtained by rotating the region bounded by the curves  $y = 2x^2$ ,  $x = 0$ ,  $x = 1$  and the  $x$ -axis about  $x$ -axis. Save your answer in a variable  $V$ .  
**Hint:** The volume of solid obtained by rotating the region bounded by the curves  $y = f(x)$ ,  $x = 0$ ,  $x = 1$  and the  $x$ -axis about the  $x$ -axis equals to  $\pi \int_0^1 [f(x)]^2 dx$ .

## Solution

```
clear all
close all
syms x
u = -x^2-5*x;
v = x^2-3;

X = solve(u==v);
x1 = X(1);
x2 = X(2);

A = int(u-v,x1,x2);

m = subs(diff(u),x,2);
c = subs(u,x,2) - m*subs(x,2);
z= m*x+c;

V = pi*int(4*x^4,0,1);
```

3. (15 marks) In this question, you need to create a function `RootTest(d_n)` to check whether an infinite series is convergent using **Root Test**.

**Remark:** Let  $s = \sum_{n=1}^{\infty} d_n$  be an infinite series. Define

$$L = \lim_{n \rightarrow \infty} |d_n|^{\frac{1}{n}}$$

- If  $L < 1$ , the series is convergent.
- If  $L > 1$ , the series is divergent.
- If  $L = 1$ , the test has no conclusion.

Given `d_n`, the  $n$ -th term of the infinite series  $s$ , check whether the series converges using root test.

- (a) If the series converges, display

The series converges.

- (b) If the series diverges, display

The series diverges.

(c) If the test has no conclusion, display

Root test has no conclusion.

Your solution should look like the sample output.

### Sample Output

```
>> syms n;RootTest((1/n)^n)
The series converges.

>> syms n;RootTest(n^n)
The series diverges.

>> syms n;RootTest((-1)^n)
Root test has no conclusion.
```

### Solution

```
function RootTest(a_n)
    syms n
    L = limit((abs(a_n))^(1/n),inf);
    if L < 1
        display('The series is convergent.')
    elseif L > 1
        display('The series is divergent.')
    else
        display('No conclusion.')
    end
```

4. (15 marks) Write a script “q4.m” to find the Taylor expansion of

$$f(x, y) = e^x \log(y)$$

at  $(x, y) = (2, 4)$  up to 4 order, where  $\log(y)$  is nature logarithm. Plot  $f(x, y)$  and its Taylor expansion in the SAME graph. You must follow the instructions as below:

### Instructions

(a) Compute the Taylor expansion of  $f(x, y) = e^x \log(y)$  at  $(x, y) = (5, 3)$  up to the order 4;

- (b) Plot the mesh of the function  $f(x, y)$  and the surface of its Taylor expansion in SAME graph, where  $x, y \in [3, 20]$ ;
- (c) Add a title as

`exp(x)log(y)` and its Taylor expansion at  $(x,y) = (5,3)$ . ;

- (d) Save your figure as `q4.jpg` by using `print('q4.jpg', '-djpeg')`.

**Note:** Please don't need to set different line style or color in these curves.

**Solution**

```
clear

syms x y
r = exp(x)*log(y);
ezmesh(r,[3 20 3 20])
hold on
ezsurf(taylor(r, [x y],[5 3],'order',4),[3 20 3 20])
title('exp(x)sin(y) and its taylor expansion at (x,y) = (5,3)')
print('q4.jpg', '-djpeg')
```

5. (20 marks) A spherical coordinate system can be seen as the three-dimensional version of the polar coordinate system. It uses three numbers  $(r, \theta, \varphi)$  to specify the position of a point in a three-dimensional space (see Figure 1).

- (a) The spherical coordinate of a point  $(r, \theta, \varphi)$  can be obtained from its Cartesian coordinate  $(x, y, z)$  by the formulae

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\varphi = \begin{cases} 0 & \text{if } x = 0 \\ \arctan \frac{y}{x} & \text{otherwise} \end{cases}$$

$$\theta = \begin{cases} 0, & \text{if } r = 0 \\ \arccos \frac{z}{r}, & \text{otherwise} \end{cases}$$

Write a function `[r, theta, phi] = cartsphe(x, y, z)` to compute the spherical coordinate  $(r, \theta, \phi)$  from a Cartesian coordinate  $(x, y, z)$ .

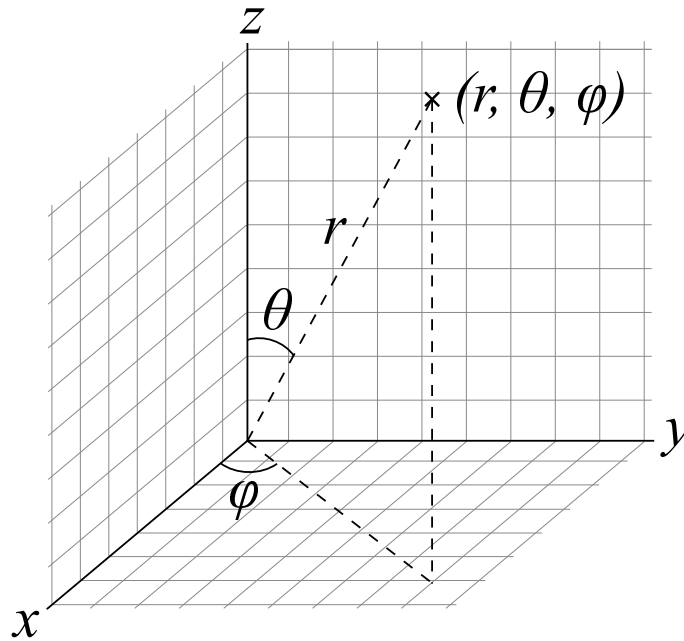


Figure 1: Spherical coordinate system

- (b) The Cartesian coordinates of a point  $(x, y, z)$  can be obtained from the spherical coordinates  $(r, \theta, \varphi)$ , where  $r \in [0, \infty)$ ,  $\theta \in [0, \pi]$ ,  $\varphi \in [0, 2\pi)$ , by

$$\begin{aligned} z &= r \cos \theta \\ y &= r \sin \theta \sin \varphi \\ x &= r \sin \theta \cos \varphi \end{aligned}$$

Write a function `[x, y, z] = sphecart(r, theta, phi)` to compute the Cartesian coordinate  $(x, y, z)$  given the spherical coordinate  $(r, \theta, \varphi)$ .



### Solution (a)

```
function [ r, theta, phi ] = cartsphe( x, y, z )

r = sqrt( x^2 + y^2 + z^2 );

if r == 0
    theta = 0;
else
    theta = acos( z / r );
end

if x == 0
    phi = 0;
else
    phi = atan(y/x);
end
end
```

### Solution (b)

```
function [ x, y, z ] = sphecart( r, theta, phi )

x = r * sin(theta) * cos( phi );
y = r * sin(theta) * sin( phi );
z = r * cos(theta);

end
```

6. (20 marks) For nonnegative integers  $n$  and  $k$ , the binomial coefficient  $C(n, k)$  is defined as

$$C(n, k) := \frac{n!}{k!(n-k)!}.$$

In this question, you need to create a MATLAB function `PrintToFile(n)` that writes some numbers into a text file. In the text file, there should be  $n+1$  lines, and the  $i^{\text{th}}$  line consists of  $i$  numbers  $C(i-1, 0)^2, C(i-1, 1)^2, \dots, C(i-1, i-1)^2$ ,  $i = 1, 2, \dots, n+1$ .

Follow the instructions below:

- (a) Use `fopen` with suitable parameters so that you can write into a file named `binomial.txt`.
- (b) Use 2 for-loops to write the numbers into the file.
- (c) Close the file using `fclose`.

**Sample Output:** For example, if you type `PrintToFile(5)` in the MATLAB Command Window, the content in `binomial.txt` should look like

```
1
1 1
1 4 1
1 9 9 1
1 16 36 16 1
1 25 100 100 25 1
```

### Solution

```
function [ ]=PrintToFile( n )

datafile = fopen('binomial.txt','w');
for i = 0:n
    for j = 0:i
        b = factorial(i)/(factorial(j)*factorial(i-j));
        fprintf(datafile,'%d ', b^2);
    end
    fprintf(datafile,'\n');
end
fclose(datafile);

end
```