MATH 2221B Mathematics Laboratory II

Lab Assignment 5

Name: ________________________ Student ID.: ________________________

In this assignment, you are asked to run `MATLAB` demos to see `MATLAB` at work. The color version of this assignment can be found in your own H:\ drive.

Instructions

1. Start `MATLAB`, until you see a window with the `MATLAB` prompt "»". This window is called the Command Window.
2. After you started have `MATLAB`, you will automatically be in the directory H:\. Please enter "`diary on`" after the `MATLAB` prompt » only once to record all your work in H:\diary. No marks will be given if no diary is found.
3. Enter "demo" after the prompt ». You will see a new window with many things to play with. This is the Demo Window.
4. In the Demo Window, try to locate figures or problems similar to those in the exercises below. Then locate the commands that generate these figures or problems. Try them in the Command Window. Just enter (or cut and paste) the commands after » to see what happens.
5. You should write your results on the lab sheet provided, and save the figures in the H: drive, in your personal drive.
6. Please read and sign the following declaration before handing in your assignment. Otherwise, no marks will be given.

I declare that the assignment here submitted is original except for source material explicitly acknowledged. I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained in the website
http://www.cuhk.edu.hk/policy/academichonesty/

________________________ ________________________
Signature Date

1 (20 marks)			
2 (20 marks)			
3 (20 marks)			
4 (20 marks)			
5 (20 marks)			

Please read the following carefully:
General Guidelines for Lab Assignment Submission.

- Please sign and date the statement of Academic Honesty.
- Please go to the class and lab indicated by your registered course code via the CUSIS system. If you go to a different lab than the one you are registered for, you will not receive credit for the assignment even if you completed it.
- Write your COMPLETE name and student ID number legibly on the cover sheet (otherwise we will not take any responsibility for your lab). Please write your answers using a black or blue pen, NOT any other color or a pencil.
- Write your solutions on a double-sided printout of this pdf file. Try to fit your answers inside the available space.
- The use of computers/cellular phones/graphing calculators/iPads will NOT be permitted during tests and lab assignments. Please do not use our lab computer to recharge your cellar phone battery. No photo taking is allowed in the lab.
- In order to make it fair for all students, during the labs and tests, if you touch/press any icons on your cellular phone, our TA will check your phone to determine whether or not you are exchanging messages with another student. If you are found cheating (in the tests or in the lab or on homework assignments), you will automatically get an F grade in this course and your act will be reported to the Department for necessary disciplinary actions.

Exercises

1. (20 marks) In linear algebra, a Hilbert matrix is a square matrix with entries being the unit fractions
$$H_{ij} = \frac{1}{i+j-1}.$$
The n-by-n Hilbert matrix A can be generated by MATLAB command `A=hilb(n)`. Please do the followings exercises and write down the commands and answers.

(a) Create a 5-by-5 Hilbert matrix A (do not need to print A).

```
A = hilb(5);
```

(b) Calculate the sum of each column of A.

```
>> sum( A )

ans =

    2.2833    1.4500    1.0929    0.8845    0.7456
```

(c) Calculate the sum of each row of A.

```
>> sum( A, 2 )

ans =

    2.2833
    1.4500
    1.0929
    0.8845
    0.7456
```

(d) Calculate the sum of the diagonal entries of A.

```
>> sum( diag(A) )

ans =

    1.7873
```

(e) Calculate the sum of the diagonal entries of R, where R is the matrix after rotating A by 90 degrees counter clockwise.

```
>> sum( diag( rot90(A) ) )

ans =

     1
```

(f) Calculate the rank of A.

```
>> rank(A)

ans =

     5
```

(g) Find the determinant of A.

```
>> det(A)

ans =

   3.7493e-12
```

(h) Find the inverse of A (do not need to print the matrix).

```
>> inv(A);
```

2. (20 marks) This exercise is about the p-norms of vectors. Let

$$\mathbf{x} = \begin{bmatrix} 5 \\ 1 \\ 0 \\ 5 \end{bmatrix}, \qquad \mathbf{y} = \begin{bmatrix} -1 \\ 0 \\ 5 \\ 10 \end{bmatrix}, \qquad \mathbf{z} = \begin{bmatrix} 8 \\ 5 \\ -1 \\ 2 \end{bmatrix}.$$

(a) Use MATLAB to compute the $1-$, $2-$, and $\infty-$norm for $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$ and write down the results.

	x	y	z
1-norm	11	16	16
2-norm	7.1414	11.2250	9.6954
∞-norm	5	10	8

(b) For each vector, sort its three norms computed above in descending order.

x: 1-norm > 2-norm > ∞-norm

y: 1-norm > 2-norm > ∞-norm

z: 1-norm > 2-norm > ∞-norm

(c) Based on (b), can you formula a conjecture regarding the ordering of these norms for any vector v? The MATLAB command `v = rand(4,1)` creates a random 4-dimensional vector `v`. Do tests on 5 random vectors to see if your conjecture is true. Write down the command for one of the 5 tests and summarize your results.

Conjecture: For any v: 1-norm $\geq$ 2-norm $\geq$ ∞-norm.

```
>>  v = rand(4,1);
    [norm(v,1), norm(v,2), norm(v,inf)]
```

```
ans =

    1.9707    1.1938    0.9378


The conjecture is true.
```

(d) Verify the triangle inequality

$$\| \mathbf{x} + \mathbf{y} \|_2 \leq \| \mathbf{x} \|_2 + \| \mathbf{y} \|_2 .$$

Write down the MATLAB command and summarize the results.

```
>> norm(x+y)

ans =

   16.3401

>> norm(x)+norm(y)

ans =

   18.3664

It satisfies the triangle inequality.
```

(e) Verify the parallelogram law

$$\| \mathbf{x} + \mathbf{y} \|_2^2 + \| \mathbf{x} - \mathbf{y} \|_2^2 = 2(\| \mathbf{x} \|_2^2 + \| \mathbf{y} \|_2^2).$$

Write down the MATLAB command and summarize the results.

```
>>     norm(x+y)^2 + norm(x-y)^2 - ...
        2*norm(x)^2-2*norm(y)^2

ans =

   1.1369e-13

It satisfies the parallelogram law.
```

3. (20 marks) In MATLAB, the eigenvectors and eigenvalues of a matrix A can be found by the command [V,D] = eig(A), where each column vector of V is an eigenvector, each diagonal entry of D is the corresponding eigenvalue and V, D satisfy

$$\mathsf{A} = \mathsf{VDV}'.$$

Let

$$\mathsf{A} = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 3 \\ 2 & 3 & 3 \end{bmatrix}.$$

(a) Find V and D.

```
>> A = [2 2 2
        2 4 3
        2 3 3];

[V,D] = eig(A)

V =

    0.4913    0.7563    0.4320
    0.3744   -0.6312    0.6793
   -0.7864    0.1720    0.5932

D =

    0.3225         0         0
         0    0.7858         0
         0         0    7.8917
```

(b) Use basic matrix operations, transpose, MATLAB built-in functions `norm` and `diag` only to check (i) whether V is an orthogonal matrix, and (ii) whether D is a diagonal matrix.
Hint: A matrix B is a zero matrix if and only if `norm(B)` is zero.

```
(i)
>> norm( V'*V - eye(3) )

ans =

   4.5827e-16

So V is a orthogonal matrix.

(ii)
>> norm( D - diag(diag(D)) )

ans =
```

```
    0
so D is a diagonal matrix.
```

(c) Check whether each column of V is an eigenvector of A.
Hint: A vector b is zero if and only if norm(b) is zero.

```
>> norm( A*V(:,1) - D(1,1)*V(:,1) )

ans =

   1.3509e-15

>> norm( A*V(:,2) - D(2,2)*V(:,2) )

ans =

   6.3049e-16
>> norm( A*V(:,3) - D(3,3)*V(:,3) )

ans =

   9.9301e-16

   So the columns of V are eigenvectors of A.
```

4. (20 marks) An $n \times n$ Vandermonde matrix has the form:

$$A = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \lambda_1 & \lambda_2 & \cdots & \lambda_n \\ \lambda_1^2 & \lambda_2^2 & \cdots & \lambda_n^2 \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_1^{n-1} & \lambda_2^{n-1} & \cdots & \lambda_n^{n-1} \end{bmatrix}.$$

Let $n = 8$, $\lambda_i = 2i$ $1 \leq i \leq 8$, and let b $=$ [1:8]'. Following the instructions given below, solve the linear systems related to A. Write down the MATLAB command and the answers.

(a) Write down the MATLAB command to create A, using basic matrix operations, MATLAB built-in functions `ones` and `linspace`. Please do not use the MATLAB build-in function `vander` or a for loop.
Hint: Think about A=B.^C. What are B and C? Write down your MATLAB commands (Do not need to print the matrix).

```
>>  B = ones(8,1)*linspace(2,16,8);
```

```
C = linspace(0,7,8)'*ones(1,8);
A = B.^C;
```

(b) Use MATLAB backslash operator (\) to solve the linear system Ax = b and compute $\|$ Ax - b $\|_2$ after you get the solution x.

```
>> x = A\b

x =

   -0.0685
    5.0475
  -10.8451
   13.8086
  -11.1279
    5.5840
   -1.5986
    0.2001

>> norm(A*x - b )

ans =

   1.3920e-08
```

(c) Use MATLAB `inv` to solve the linear system Ax = b and compute $\|$ Ax - b $\|_2$ after you get the solution x.

```
>> x = inv(A)*b

x =

   -0.0685
    5.0475
  -10.8451
   13.8086
  -11.1279
    5.5840
   -1.5986
    0.2001

>> norm( A*x - b )

ans =

   1.7989e-07
```

5. (20 marks) A system of linear equations in matrix form $\mathsf{A}\mathrm{x} = \mathrm{b}$ can be represented by the augmented matrix $[\mathsf{A}|\mathbf{b}]$ (in MATLAB $[\mathsf{A}\ \mathrm{b}]$). If there is at least one solution, a system of equations is called consistent. A system of equations that has no solutions is said to be inconsistent. Determining whether a system is consistent or inconsistent can be done by looking at the rank of the augmented matrix and the rank of the matrix of coefficients A.

Let n be the number of unknowns. Few important cases are:

(i) If the augmented matrix and the matrix A both have rank r, then the system is consistent. If $\mathsf{r} = \mathsf{n}$, then there is one solution. If $\mathsf{r} < \mathsf{n}$, there are infinitely many solutions and the difference n - r determines the number of parameters in the family of solutions.

(ii) If the augmented matrix and the matrix A do not have the same rank, then the system is inconsistent.

The row-reduced echelon form of a matrix can be computed by using MATLAB function reff.

(a) Consider the following linear system:

$$\begin{array}{rcrcrcrcr} 7x & - & y & - & 11z & + & 2w & = & 1 \\ 5x & - & 6y & - & 5z & + & 32w & = & -8 \\ -33x & + & 14y & + & z & - & 4w & = & 4 \\ -2x & & & - & 10z & + & 6w & = & -86 \end{array}$$

i. Find the reduced row echelon form for the above system. Write down the MATLAB commands and results.

```
>> A = [7 -1 -11 2
        5 -6 -5 32
        -33 14 1 -4
        -2 0 -10 16];
    b = [1 -8 4 -6]';

    rref([A,b])

ans =

    1.0000         0         0         0    4.5886
         0    1.0000         0         0   11.3797
         0         0    1.0000         0    2.0654
         0         0         0    1.0000    1.4895
```

ii. Based on the reduced row echelon form, find the rank of A and also the rank of the augmented matrix. Determine whether the system is consistent or inconsistent. If the system is consistent, find its solution(s) using the reduced row echelon form.

```
The rank of A is 4, because the first 4 columns of
the reduced row echelon form is of rank 4;
The rank of [A,b] is 4, because
the reduced row echelon form is of rank 4;

Since the rank of A and the rank of
the augmented matrix are the same, the system
is consistent.

The solution is (4.5886   11.3797     2.0654    1.4895).
```

(b) Consider the following linear system:

$$\begin{array}{rcrcrcrcr} 5x & + & 3y & - & z & - & 2w & = & 3 \\ 5x & - & 3y & - & z & + & 16w & = & -1 \\ 2x & + & 11y & - & z & + & 2w & = & -1 \\ 1x & + & 2 & + & 4z & - & 4w & = & -4 \\ 4x & - & 4y & + & 5z & - & 3w & = & 5 \end{array}$$

i. Find the reduced row echelon form for the above system. Write down the MATLAB commands and results.

```
>> A = [5 3 -1 -2;
        5 -3 -1 16;
        2 11 -1 2;
        1 2 4 -4;
        4 -4 5 -3];
    b = [3; -1; -1; -4; 5];
>> rref([A b])

ans =

     1     0     0     0     0
     0     1     0     0     0
     0     0     1     0     0
     0     0     0     1     0
     0     0     0     0     1
```

ii. Based on the reduced row echelon form, find the rank of A and also the rank of the augmented matrix. Determine whether the system is consistent or inconsistent. If the system is consistent, find its solution(s).

```
The rank of A is 4, because the first 4 columns of
the reduced row echelon form is of rank 4;
```

```
The rank of [A,b] is 5, because
the reduced row echelon form is of rank 5;

Since the rank of A and the rank of
the augmented matrix are not the same,
the system is inconsistent.
```