MATH 2221B Mathematics Laboratory II

Lab Assignment 11

Name: _____   Student ID.: _____

In this assignment, you are asked to run MATLAB demos to see MATLAB at work. The color version of this assignment can be found in your own **H:** drive.

Instructions

1. Start MATLAB, until you see a window with the MATLABprompt ">>". This window is called the **Command Window**.

2. After you started have MATLAB, you will automatically be in the directory **H:**. Please enter "diary on" after the MATLABprompt >> only once to record all your work in H:\diary. No marks will be given if no diary is found.

3. You should write your results on the lab sheet provided, and save the figures in the **H:** drive, in your personal drive.

4. Please read and sign the following declaration before handing in your assignment. Otherwise, no marks will be given.

I declare that the assignment here submitted is original except for source material explicitly acknowledged. I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained on the website http://www.cuhk.edu.hk/policy/academichonesty/

_____   _____
Signature   Date

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1 (25 marks)		3 (25 marks)	
2 (25 marks)		4 (25 marks)	
		Total	/100 pts

Please read the following carefully:
General Guidelines for Lab Assignment Submission.

- Please sign and date the statement of Academic Honesty.

- Please go to the class and lab indicated by your registered course code via the CUSIS system. If you go to a different lab than the one you are registered for, you will not receive credit for the assignment even if you completed it.

- Write your COMPLETE name and student ID number legibly on the cover sheet (otherwise we will not take any responsibility for your lab). Please write your answers using a black or blue pen, NOT any other color or a pencil.

- Write your solutions on a double-sided printout of this pdf file. Try to fit your answers inside the available space.

- The use of computers/cellular phones/graphing calculators/iPads will NOT be permitted during tests and lab assignments. Please do not use our lab computer to recharge your cellar phone battery.

- In order to make it fair for all students, during the labs and tests, if you touch/press any icons on your cellular phone, our TA will check your phone to determine whether or not you are exchanging messages with another student. If you are found cheating (in the tests or in the lab or on homework assignments), you will automatically get an F grade in this course and your act will be reported to the Department for necessary disciplinary actions.

Exercises

Note: Remember to add ";" if you want to suppress the output. Marks will be
deducted if there is any unnecessary output. Unless specified in the
question, you should not print anything on the screen.

1. (25 marks) Consider a particle moving in a two dimensional space with acceleration

$$a(t) = -10\mathbf{j}$$

Write a script `q1.m` to complete the following tasks.

(a) Let t be a symbolic variable. Compute $s(t)$ and $v(t)$, where $s(t), v(t)$
are the displacement and velocity of the particle at time t respectively.
Assume

$$s(0) = 0\mathbf{i} + 0\mathbf{j} \qquad v(0) = 20\mathbf{i} + 10\mathbf{j}$$

Hint:

- $v_i(t) = v_i(0) + \int a_i(r)\,dr$ and $s_i(t) = s_i(0) + \int v_i(r)\,dr$, $i = 1, 2$. Here
  $v_i(t), s_i(t)$ are the i-th element of $s(t), v(t)$ respectively.
- You may define $a = [0, -10]$ and take integration with respect to t.

(b) Compute the velocity at $t = 0.5$, and **print your answer on the screen**.

(c) At what time the particle attains its highest point (The velocity in the
vertical direction equals to zero)? Compute it using the function `solve`.

(d) At what time the y-coordinate of the particle becomes zero? Compute it
using the function `solve`.

Solution

```
clear all
close all

syms t
a = [0,-10];
v = [20,10] + int(a,t);
s = [0,0] + int(v,t);

v2 = subs(v,0.5)
t0 = solve(v(2)==0);
t1 = solve(s(2)==0);
```

2. (25 marks) In this question, you need to check whether an infinite series is convergent by using *Integral test*.

Remark: Suppose $f(x)$ is a continuous, positive and decreasing function on $[1, \infty]$, and $f(n) = a_n$, then

- If $\int_1^\infty f(x)dx < \infty$, then $\sum_{n=1}^\infty a_n$ is convergent.
- Otherwise, then $\sum_{n=1}^\infty a_n$ is divergent.

Write a function `IntegralTest(a_n)` to check whether an infinite series $s = \sum_{n=1}^\infty a_n$ is convergent. If it is convergent, display

```
The series is convergent.
```

Here `l` is the limit of the series. If it is divergent, display

```
The series is divergent.
```

Sample Output

```
>> IntegralTest(1/n^2);
The series is convergent.
>> IntegralTest(1/n);
The series is divergent.
```

Write down the output of the following:

```
>> IntegralTest(n^2*exp(-n^3));
```

Solution

```
function IntegralTest(a_n)
    integral = int(a_n,1,inf);
    if isfinite(integral)
        display('The series is convergent.')
    else
        display('The series is divergent.')
    end
end
```

3. (25 marks) Write a script `q3.m` to complete the following tasks:

   (a) Define a symbolic variable `x`.

   (b) Set $y1 = x$, $y2 = x^2$

   (c) Compute the x-coordinate of the intersection points `x1,x2`, where `x1<x2`.

   (d) Find the area between these two curves, and display

   > `The area between these two curves is A`

   Here `A` is a numerical value computed by you.

   (e) This part is independent of previous parts. Use `solve` to check whether the following systems of equations have solutions or not. You just need to write down **YES** or **NO** for each part in the space provided below.

   Hint: You may formulate the system as $Ax = b$, then you can use `solve` to finish the problems.

   i. $\begin{cases} 7z + 5y - 3z = 12 \\ 3x - 5y + 2z = -8 \\ 5x + 3y - 7z = 0 \end{cases}$

   ii. $\begin{cases} x + y + z = 6 \\ 3x + 3y + 3z = 1 \\ 5x + 3y - 7z = 0 \end{cases}$

   Remark: If there exists solution(s), The three planes have intersection point(s).

Part(e) answer

```
(i) Yes, since solve(lhs==rhs) has solution for x,y and z
(ii) No, since solve(lhs==rhs) does not have solution for x,y and z
```

Solution

```
clear all
close all

syms x y z
y1 = x;
y2 = x^2;
X = solve(y1==y2);
x1 = X(1);
x2 = X(2);
A = double(int(y1-y2,x1,x2));
display(['The area between these two curves is ' num2str(A)])

coeff = [7,5,-3;3,-5,2;5,3,-7];
var = [x;y;z];

lhs = coeff*var;
rhs = [12;-8;0];
solve(lhs==rhs);

coeff = [1,1,1;3,3,3;5,3,-7];
var = [x;y;z];

lhs = coeff*var;
rhs = [6;1;0];
solve(lhs==rhs);
```

4. (25 marks) In this question, you have to write a function `TaylorSeries(f)` to finish the following tasks.

   (a) Given a symbolic variable `f`, create a symbolic variable `g` such that

   $$g(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots \frac{f^{(n)}(0)}{20!}x^{20}$$

   Here x is a symbolic variable.

   (b) Compute the $g(0.1)$, and display the following sentence:

   ```
   Using 21-terms Taylor series to approximate f, f = l
   ```

   Here `l=g(0.1)`

Sample Output

```
>> TaylorSeries(1/(1-x))
Using 21-terms Taylor series to approximate f, f = 1.1111
>> TaylorSeries(1/(1+x))
Using 21-terms Taylor series to approximate f, f = 0.90909.
```

Write down the output of the following:

```
>> TaylorSeries(cos(x))
```

Solution

```
function TaylorSeries(f)
    syms y x
    g = subs(f,0);
    for i = 1:20
        g = g + subs(diff(f,i),0)*x^i/factorial(i);
    end
    d = double(subs(g,0.1));
    display(['Using 21-terms Taylor series to approximate f, f = ' num2str(d))
end
```