Assignment No. 1. Due date: Oct 31, 2018.

Q1. Suppose n is even, use quadratic polynomials to get an upper bound of $\max_{\lambda \in [a.b]} |P(\lambda)|$. And hence find a bound for $\dfrac{E_Q^2(\mathbf{x}_k)}{E_Q^2(\mathbf{x}_0)}$.

Q2. Let $A, E \in \mathbb{C}^n$ be Hermitian with $\operatorname{rank}(E) = 1, E \leq 0$. Show that

$$\lambda_{k-1}(A) \leq \lambda_k(A + E) \leq \lambda_k(A), \text{ for } 2 \leq k \leq n$$

$$\lambda_1(A) + \lambda_1(E) \leq \lambda_1(A + E) \leq \lambda_1(A)$$

Q3. Let $A, E \in \mathbb{C}^n$ be Hermitian with $\operatorname{rank}(E) = p$. Show that there exist at most p eigenvalues of $A + E$ outside $[\lambda_1(A), \lambda_n(A)]$.

Q4. Find the eigenvalues and eigenvectors of the Laplacian matrix with Neumann Boundary Condition.

Q5. Complete the second half of the proof of Courant Fisher min-max Theorem, i.e.

$$\lambda_k(A) = \max_{\dim(K)=n-k+1} \min_{\mathbf{x} \in K \setminus \{\mathbf{0}\}} \frac{\mathbf{x}^* A \mathbf{x}}{\mathbf{x}^* \mathbf{x}}$$