Observation I

(1) All ERO are recorded in last m columns

$$A = [R \mid I]$$

$E_\ell \cdots E_2 E_1$ at ith iteration

$$[U \mid V]$$

$$\underbrace{E_\ell \cdots E_1}_{E} A = [U \mid V]$$

$$EA = [U \mid V]$$

$$\| $$

$$E[R \mid I]$$

$$\|$$

$$[ER \mid E]$$

$$\boxed{E = V}$$

Observation ii) $E^{-1} = B_i$ your current basic matrix.

(In particular, your B_i is invertible)

pf Tableau i $\quad I = (\vec{e}_1, \vec{e}_2, \dots, \vec{e}_m) = \left(\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix}, \dots \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \right)$

$$T_i = \left(\begin{array}{ccccccccccc} * & * & \vec{e}_{i_1} & * & * & \vec{e}_{i_2} & \cdots * & \vec{e}_{i_j} & \cdots * & \vec{e}_{i_m} & * \\ & & \uparrow & & & \uparrow & & \uparrow & & \uparrow & \\ & & x_{i_1} & & & x_{i_2} & & x_{i_j} & & x_{i_m} & \end{array} \right)$$

$$\underbrace{x_{i_1} \quad x_{i_2} \quad x_{i_j} \quad x_{i_m}}_{\text{basic variables}}$$

$B = (\vec{a}_{i_1}\ \vec{a}_{i_2} \cdots \vec{a}_{i_m})$

$$\underbrace{E_\ell \cdots E_1}_{E} A = T_i$$

$$A = E^{-1} (\quad T_i \quad) \qquad (1)$$

$$(\vec{a}_1\ \vec{a}_2 \cdots \vec{a}_n) = F (* * \vec{e}_{i_1} * * \vec{e}_{i_2} \cdots * \vec{e}_{i_m} *)$$

$$= \cancel{(\vec{f}_1\ \vec{f}_2 \cdots \vec{f}_m)}$$

$$= (* * \vec{f}_{i_1} * \cdots * \vec{f}_{i_2} \cdots \vec{f}_{i_m} *)$$

$$E^{-1} = F = (\vec{f}_1 \cdots \vec{f}_n) = (\vec{a}_{i_1}, \vec{a}_{i_2}, \dots \vec{a}_{i_m}) = B.$$

Corollary $\quad \underbrace{A = BT_i \qquad A = BI}$

$$\Downarrow$$

$$T_i = I \quad \#$$

<u>Observation (iii)</u> $T_1 = [\, R \mid I \mid \vec{b} \,]$

at Tableau i

$$T_i = [\, Y_i \mid \vec{y}_0 \,]$$

$\Rightarrow$ $\vec{y}_0 = \vec{X}_{B_i}$ the solution corresponding to the current basic matrix B_i.

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pf: $A = (\underbrace{B_i}_{\text{Basic matrix (nonsingular)}} \mid R_i)$

$$(B_i \mid R_i)\begin{pmatrix} \vec{X}_{B_i} \\ \vec{0} \end{pmatrix} = \vec{b} \iff \boxed{B_i \vec{X}_{B_i} = \vec{b}}$$

$$\iff \boxed{\vec{X}_{B_i} = \underset{\substack{\parallel \\ \vec{y}_0\ (\text{N.T.P})}}{B_i^{-1}\vec{b}}} \ ?$$

$$\underbrace{E_r \cdots E_1}_{\parallel}(A \mid \vec{b}) = T_i = (Y_i \mid \vec{y}_0)$$

$$E(A \mid b) = E(A \mid b) = (EA \mid E\vec{b})$$

$E\vec{b} = \vec{y}_0$

$\parallel$

$B_i^{-1}\vec{b}$ ($\because$ observation (ii))

$\parallel$ #

$\vec{X}_{B_i}$

① finding a starting BFS

Simplex Algorithm

① optimality condition. entering variable

$0 \rightsquigarrow \neq 0$

non-Basic basic

② feasibility condition. leaving variable

$\neq 0 \rightsquigarrow 0$

basic non-basic

③ move by ERO.
elementary row operations
(Gaussian Elimination)