Homework 5 of Math 3210

Due Date: December 11, before 18:00. Please put your homework in the assignment box before the due time. The box is on the 2nd floor of Lady Shaw Building opposite to Rm 221. The solution to the homework problems will be uploaded to the course webpage right after the due time. Because of this, *any late homework submission will not be accepted.*

1. Consider the following LPP.
Maximize $z = 120x_1 + 100x_2$ subject to

$$\begin{cases} 2x_1 + 2x_2 + x_3 & = 8, \\ 5x_1 + 3x_2 + x_4 & = 15, \\ x_1, x_2, x_3, x_4 & \geq 0. \end{cases}$$

(i) Solve the LPP using dual simplex method to obtain a basic feasible solution $\mathbf{x}_B$.
(ii) If we change the vector $\mathbf{b} = (8, 15)^T$ by $\Delta\mathbf{b} = (\Delta b_1, \Delta b_2)^T$ (i.e. the new $\mathbf{b}$ is given by $\mathbf{b}+\Delta\mathbf{b}$), determine the range of $\Delta\mathbf{b}$ in order to keep $\mathbf{x}_B$ feasible.
(iii) Let $\Delta b_1 = 1$ and $\Delta b_2 = -1$, solve the new problem starting from the optimal simplex tableau obtained in (i).

2. Consider the following LPP:
maximize $z = x_1 - 2x_2 - 3x_3 - x_4 - x_5 + 2x_6$ subject to

$$\begin{cases} x_1 + 2x_2 + 2x_3 + x_4 + x_5 & = 12, \\ x_1 + 2x_2 + x_3 + x_4 + 2x_5 + x_6 & = 18, \\ 3x_1 + 6x_2 + 2x_3 + x_4 + 3x_5 & = 24, \\ x_1, \cdots, x_6 & \geq 0. \end{cases}$$

(i) Solve the above LPP by dual simplex method.
(ii) Suppose we change $\mathbf{c} = (1, -2, -3, -1, -1, 2)^T$ by $\Delta\mathbf{c} = (-1, 1, 0, -\frac{1}{2}, 1, 2)^T$, solve the new problem starting from the optimal simplex tableau obtained in (i).

3. Consider the LPP:
minimize $z = -x_1 - x_2 + x_3 + x_4$ subject to

$$\begin{cases} x_1 + x_3 & = 1, \\ x_2 + x_4 & = 2, \\ x_1, x_2, x_3, x_4 & \geq 0. \end{cases}$$

Let $\mathbf{x}_k = (0.8, 0.1, 0.2, 1.9)^T$.
(i) Compute the current value of z with respect to $\mathbf{x}_k$.
(ii) Find a rescaled problem of the above LPP such that $\mathbf{e} = (1, 1, 1, 1)^T$ is feasible.
(iii) Find the projected gradient vector $\mathbf{d}_k$ and normalize it so that $\|\mathbf{d}_k\| = 1$.
(iv) Let $\widehat{\mathbf{x}}_{k+1} = e + \alpha \mathbf{d}_k$. Find an upper bound α_{max} such that $\widehat{\mathbf{x}}_{k+1}$ is feasible. Choosing $\alpha = 0.9\alpha_{max}$, compute the value of the unscaled solution $\mathbf{x}_{k+1}$.

4. Consider the LPP:
minimize $z = -2x_1 + x_2$ subject to

$$\begin{cases} x_1 - x_2 & \leq 15, \\ x_2 & \leq 15, \\ x_1, x_2 & \geq 0. \end{cases}$$

(i) Introduce slack variables x_3 and x_4 to covert the LPP to its standard form.
(ii) Apply Karmakar's interior point method to the problem with the trial solution $\mathbf{x}_0 = (16.9, 2, 0.1, 13)^T$. Scale the solution $\mathbf{x}_0$ to $\mathbf{e} = (1, 1, 1, 1)^T$ first and choose $\alpha = 0.9\alpha_{max}$ and do one iteration step, where α_{max} is the same as in part (iv) of Q3.

5. One of management's goals in a goal programming problem is expressed algebraically as

$$3x_1 + 4x_2 + 2x_3 = 60,$$

where 60 is the specific numeric goal and the left-hand side gives the level achieved toward meeting this goal.
(i) Letting y^+ be the amount by which the level achieved exceeds this goal (if any) and y^- be the amount under the goal (if any), show how this goal would be expressed as an equality constraint when reformulating the problem as a linear programming model.
(ii) If each unit over the goal is considered twice as serious as each unit under the goal, what is the relationship between coefficients of y^+ and y^- in the objective function being minimized in this linear origramming model?

6. Mangement of the Albert Franko Co. has established goals for the market share it wants each of the company's two new products to capture in their respective markets. Specifically, mangement wants Product 1 to capture at least 15 percent of its market and Product to to capture at least 10 percent of its market. Three advertising campaigns are being planed to try to achieve these market shares. One is targeted diectly on the first product. The second targets the second product. The third is intended to enhance the general reputation of the company and its products. Letting x_1, x_2 and x_3 be the amount of money allocated (in millions of dollars) to these respective campaigns, the resulting market share (expressed as a percentage) for the two products are estimated to be

$$\text{Market share for Product 1} = 0.5x_1 + 0.2x_3,$$

$$\text{Market share for Product 2} = 0.3x_2 + 0.2x_3.$$

A total of \$55 million is available for the three advertising campaigns, but management wants at least \$10 million devoted to the third campaign. If both market share goals can not be achieved, management considers each 1 percent decrease in the market share from the goal to be equally serious for the two products. In this light, management wants to know how to most effectively allocate the available money to the three campaigns.

(i) Formulate a goal programming model for this problem.

(ii) Reformulate this model as a linear programming model.

(iii) Use simplex method to solve this model.