## Crystals and Crystalline Cohomology

## Dr. Lei Zhang

## Exercise sheet $5^1$

**Exercise 1.** Let  $\mathcal{C}$  be a site, and let  $\tilde{\mathcal{C}}$  be the associated topos. Let  $(\mathcal{C}, \mathcal{O})$  be a ringed topos, and let  $\mathcal{F}$  be an  $\mathcal{O}$ -module. We say  $\mathcal{F}$  is quasicoherent if for any object  $U \in \mathcal{C}$  there exists a covering  $\{U_i \to U\}_{i \in I}$  such that there is an exact sequence

$$\mathcal{O}|_U^{\oplus I} \to \mathcal{O}|_U^{\oplus J} \to \mathcal{F}|_U \to 0$$

Show that if we take C to be the crystalline site  $\operatorname{Cris}(X/S)$ , then an  $\mathcal{O}_{X/S}$ -module  $\mathcal{F}$  is quasi-coherent if and only if it is a quasi-coherent crystal.

 $<sup>^1\</sup>mathrm{If}$  you have any questions concerning these exercises you can contact me via <code>l.zhang@fu-berlin.de</code>.