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Crystals and Crystalline Cohomology

Dr. Lei Zhang

Exercise sheet 4[1]

Exercise 1. Let (A, I, γ) be a PD-triple with I principal. Let B be an A-algebra. Then there exists a unique PD-structure on IB so that the map $(A, I, \gamma) \to (B, IB, \delta)$ is a PD-morphism.

Exercise 2. Let X be a site, and let $\tilde{X}$ be the corresponding topos. Show that the presheaf which associate with each object of X the set with exactly one element is a sheaf on X, and that this sheaf is the terminal object of $\tilde{X}$.

Exercise 3. Let $\mathcal{T}$ be a topos, and let $T \in \mathcal{T}$. Check that the functor $\Gamma(T, -) \colon \mathcal{T} \longrightarrow ((\text{Sets}))$ sending $F \mapsto \operatorname{Hom}_{\mathcal{T}}(T, F)$ is left exact.

Exercise 4. In the class we defined the morphism of topoi

$$u_{X/S} \colon (X/S)_{\text{Cris}} \longrightarrow X_{\text{Zar}}$$

by the rule

(1) for any $\mathcal{F} \in (X/S)_{\text{Cris}}$ and $j \colon U \to X$ open embedding we define
$$u_*(\mathcal{F})(U) := \Gamma((U/S)_{\text{Cris}}, \mathcal{F}|_U)$$
(2) for any $E \in X_{\text{Zar}}$ and $(U, T, \delta) \in \text{Cris}(X/S)$ we set
$$(u^*(E))(U, T, \delta) := E(U)$$

Check that our formulas really make $u_*(\mathcal{F})$ and u^*E sheaves on the corresponding site.

[1]If you have any questions concerning these exercises you can contact me via l.zhang@fu-berlin.de.