December 4, 2017

Crystals and Crystalline Cohomology

Dr. Lei Zhang

Exercise sheet 3[1]

Exercise 1. Let B be a ring. Let $d_B\colon B \to \Omega_{B/\mathbb{Z}}$ be the differential. Let M be the free B-module generated on the free basis $\{db | b \in B\}$. Show that there exists a unique map of abelian groups $M \to \Omega^2_{B/Z}$ sending $adb \mapsto da \bigwedge db$, where $a, b \in B$.

Exercise 2. In the class we have defined the categoris $\mathrm{CRIS}(X/S)$ and $\mathrm{Cris}(X/S)$ and the notion of coverings of an object. Check that with the so defined covering $\mathrm{CRIS}(X/S)$ and $\mathrm{Cris}(X/S)$ become sites.

Exercise 3. Suppose that we have a PD-morphism $(S, I, \gamma) \to (S', I', \gamma')$ and a diagram:

$$\begin{array}{ccc} X & \longrightarrow & X' \\ \downarrow & & \downarrow \\ S_0 & \longrightarrow & S_0' \end{array}$$

where $S_0 = \mathrm{Spec}\,(\mathcal{O}_S/I)$. Then we have an obvious functor

$$f\colon \mathrm{CRIS}(X/S) \longrightarrow \mathrm{CRIS}(X'/S')$$

Show that the functor f is both continuous and cocontinuous, hence it induces a map between topoi

$$(X/S)_{\mathrm{CRIS}} \xrightarrow{f_{\mathrm{CRIS}}} (X'/S')_{\mathrm{CRIS}}$$

[1] If you have any questions concerning these exercises you can contact me via l.zhang@fu-berlin.de.