## Crystals and Crystalline Cohomology

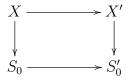
## Dr. Lei Zhang

## Exercise sheet $3^1$

**Exercise 1.** Let *B* be a ring. Let  $d_B: B \to \Omega_{B/\mathbb{Z}}$  be the differential. Let *M* be the free *B*-module generated on the free basis  $\{db|b \in B\}$ . Show that there exists a unique map of abelian groups  $M \to \Omega_{B/Z}^2$ sending  $adb \mapsto da \wedge db$ , where  $a, b \in B$ .

**Exercise 2.** In the class we have defined the categoris  $\operatorname{CRIS}(X/S)$  and  $\operatorname{Cris}(X/S)$  and the notion of coverings of an object. Check that with the so defined covering  $\operatorname{CRIS}(X/S)$  and  $\operatorname{Cris}(X/S)$  become sites.

**Exercise 3.** Suppose that we have a PD-morphism  $(S, I, \gamma) \rightarrow (S', I', \gamma')$  and a diagram:



where  $S_0 = \text{Spec}(\mathcal{O}_S/I)$ . Then we have an obvious functor

$$f: \operatorname{CRIS}(X/S) \longrightarrow \operatorname{CRIS}(X'/S')$$

Show that the functor f is both continuous and cocontinuous, hence it induces a map between topoi

$$(X/S)_{\text{CRIS}} \xrightarrow{f_{\text{CRIS}}} (X'/S')_{\text{CRIS}}$$

 $<sup>^1\</sup>mathrm{If}$  you have any questions concerning these exercises you can contact me via l.zhang@fu-berlin.de.