

THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MMAT5520
Differential Equation & Linear Algebra

Assignment 2

Due date: Oct 14 (Tue)

Exercise 2.1:

1. Find the reduced row echelon form of the following matrices.

$$(f) \begin{pmatrix} 1 & -2 & -4 & 5 \\ -2 & 4 & -3 & 1 \\ 3 & -6 & -1 & 4 \end{pmatrix}$$

$$(g) \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 2 & 3 & 4 \\ -1 & -2 & -1 & -2 & -3 \end{pmatrix}$$

2. Solve the following systems of linear equations.

$$(c) \begin{cases} 2x_1 - x_2 + 5x_3 = 15 \\ x_1 + 3x_2 - x_3 = 4 \\ x_1 - 4x_2 + 6x_3 = 11 \\ 3x_1 + 9x_2 - 3x_3 = 12 \end{cases}$$

$$(e) \begin{cases} x_1 - 2x_2 + x_3 + x_4 = 1 \\ x_1 - 2x_2 + x_3 - x_4 = -1 \\ x_1 - 2x_2 + x_3 + 5x_4 = 5 \end{cases}$$

Exercise 2.2:

3. Let \mathbf{A} be a square matrix. Prove that \mathbf{A} can be written as the sum of a symmetric matrix and a skew-symmetric matrix.

Exercise 2.3:

1. Find the inverse of the following matrices.

$$(b) \begin{pmatrix} 5 & 7 \\ 4 & 6 \end{pmatrix}$$

$$(e) \begin{pmatrix} 1 & -3 & -3 \\ -1 & 1 & 2 \\ 2 & -3 & -3 \end{pmatrix}$$

2. Solve the following systems of equations by finding the inverse of the coefficient matrices.

$$(b) \begin{cases} 5x_1 + 3x_2 + 2x_3 = 4 \\ 3x_1 + 3x_2 + 2x_3 = 2 \\ \quad \quad \quad x_2 + x_3 = 5 \end{cases} .$$

Exercise 2.4:

1. Evaluate the following determinants.

$$(c) \begin{vmatrix} 5 & 3 & 0 & 6 \\ 4 & 6 & 4 & 12 \\ 0 & 2 & -3 & 4 \\ 0 & 1 & -2 & 2 \end{vmatrix}$$

3. For the given matrix \mathbf{A} , evaluate \mathbf{A}^{-1} by finding the adjoint matrix $\text{adj}\mathbf{A}$ of \mathbf{A} .

$$(b) \mathbf{A} = \begin{pmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{pmatrix}$$

4. Use Cramer's Rule to solve the following linear systems.

$$(a) \begin{cases} 4x_1 - x_2 - x_3 = 1 \\ 2x_1 + 2x_2 + 3x_3 = 10 \\ 5x_1 - 2x_2 - 2x_3 = -1 \end{cases}$$

Exercise 2.5:

1. Find the equation of the parabola of the form $y = ax^2 + bx + c$ passing through the given set of three points.

$$(a) (0, -5), (2, -1), (3, 4)$$

2. Find the equation of the circle passing through the given set of three points.

$$(a) (-1, -1), (6, 6), (7, 5)$$