**THE CHINESE UNIVERSITY OF HONG KONG**
**Department of Mathematics**
**MATH1010D&E (2016/17 Term 1)**
**University Mathematics**
**Tutorial 8**

**Indefinite Integral** A **primitive** / **anti-derivative** of a continuous function $f:(a,b)\to\mathbb{R}$ is a differentiable function $F:(a,b)\to\mathbb{R}$ such that

$$F'(x)=f(x)\quad\text{for any } x\in(a,b).$$

**Remark:** Primitives are not unique: any two primitives of $f$ must differ by a constant. The **indefinite integral** of $f$ is the collection of all primitives of $f$, denoted by $\int f(x)dx$. We shall write

$$\int \underbrace{f(x)}_{\text{integrand}}\,dx=\underbrace{F(x)}_{\text{primitve}}+\underbrace{C}_{\substack{\text{integration}\\\text{contsant}}}.$$

**Operations on infinite integrals** Let $f,g:(a,b)\to\mathbb{R}$ be continuous and $k\in\mathbb{R}$. Then

$$1)\int(f(x)+g(x))dx=\int f(x)dx+\int g(x)dx;\quad\text{and}\quad 2)\int kf(x)dx=k\int f(x)dx.$$

**Change of variables** Let $f:[a,b]\to\mathbb{R}$ be continuous, $\varphi$ be differentiable and $g$ be continuous such that $f(x)=g(\varphi(x))\varphi'(x)$ for any $x\in(a,b)$. Then

$$\int f(x)dx=\int g(\varphi(x))\varphi'(x)dx=\int g(u)du.$$

**Problems that may be demonstrated in class** :

Q1. Evaluate $\int f(x)dx$ for different functions $f(x)$ as below:

(a) $x\sqrt{x-4}$; (b) $\frac{2x-5}{x^2-5x+36}$; (c) $3^{e^x+1}e^x$; (d) $\sin^2 x$;

(e) $\sin^3 x$; (f) $\tan x$; (g) $\tan^2 x$; (h) $\tan^3 x$;

(i) $\sec x$; (j) $\frac{1}{\sqrt{x^2+4}}$; (k) $\sin 7x\cos 4x$; (l) $\frac{\arctan x}{x^2+1}$.

Q2. In this question, we study the behaviour of the indefinite integral $\int\frac{P(x)}{ax^2+bx+c}dx$, where $P(x)$ is a polynomial and $a,b,c\in\mathbb{R}$ with $a\neq 0$.

(a) Find the discriminant of the equation $x^2-5x+6=0$. Find real constants $A$ and $B$ such that $\frac{x-5}{x^2-5x+6}\equiv\frac{A}{x-2}+\frac{B}{x-3}$. Hence evaluate $\int\frac{x-5}{x^2-5x+6}dx$.

(b) Find the discriminant of the equation $x^2+2x+1=0$. Find real constants $A$ and $B$ such that $\frac{3x+2}{x^2+2x+1}\equiv\frac{A}{x+1}+\frac{B}{(x+1)^2}$. Hence evaluate $\int\frac{3x+2}{x^2+2x+1}dx$.

(c) Find the discriminant of the equation $x^2+2x+2=0$. Evaluate $\int\frac{2x+3}{x^2+2x+2}dx$.

(d) Find real constants $A,B,C,D$ such that $\frac{x^3-3x^2-3x+7}{x^2-5x+6}\equiv Ax+B+\frac{Cx+D}{x^2-5x+6}$. Hence evaluate $\int\frac{x^3-3x^2-3x+7}{x^2-5x+6}dx$.

Q3. Let $t=\tan\frac{\theta}{2}$. Express $\sin\theta$ and $\cos\theta$ in terms of $t$. Evaluate $\int\frac{d\theta}{\sin\theta-\cos\theta-1}$.