Tutorial 6.

Chapter 15 Practice Exercises.

27. $V=\int_{-\pi/2}^{\pi/2}\int_{-\cos y}^{0}\int_{0}^{-2x} dz\,dx\,dy = \int_{-\pi/2}^{\pi/2}\int_{-\cos y}^{0} -2x\,dx\,dy = \int_{-\pi/2}^{\pi/2} -x^2\Big|_{-\cos y}^{0} dy$

$= \int_{-\pi/2}^{\pi/2} \cos^2 y\,dy = \int_{-\pi/2}^{\pi/2} \frac{\cos 2y+1}{2}\,dy = \frac{\pi}{2}$

36. (a) Bounded on the top and bottom by the sphere $x^2+y^2+z^2=4$, on the right by the right circular cylinder $(x-1)^2+y^2=1$, on the left by the plane $y=0$

(b) $\begin{cases} 0<x<2 \\ 0\le y\le \sqrt{2x-x^2} \\ -\sqrt{4-x^2-y^2}\le z\le\sqrt{4-x^2-y^2}\end{cases} \Leftrightarrow \begin{cases} 0\le r\cos\theta\le 2 \\ 0\le r\sin\theta\le\sqrt{2r\cos\theta-r^2\cos^2\theta} \\ -\sqrt{4-r^2}<z\le\sqrt{4-r^2}\end{cases} \Leftrightarrow \begin{cases} \cos\theta\ge 0,\ 0<r\le\frac{2}{\cos\theta} \\ \sin\theta\ge 0,\ 0\le r\le 2\cos\theta\le 2\le\frac{2}{\cos\theta} \\ -\sqrt{4-r^2}\le z\le\sqrt{4-r^2}\end{cases}$

$\Leftrightarrow \begin{cases} 0\le\theta\le\pi/2 \\ 0\le r\le 2\cos\theta \\ -\sqrt{4-r^2}\le z\le\sqrt{4-r^2}\end{cases}$

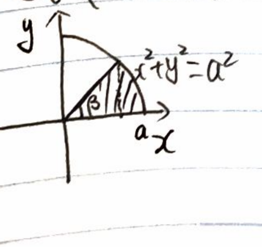
Thus, $\int_0^2\int_0^{\sqrt{2x-x^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} dz\,dy\,dx = \int_0^{\pi/2}\int_0^{2\cos\theta}\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} dz\,r\,dr\,d\theta$

Chapter 15 Additional and Advanced Exercises.

12. (a) $\begin{cases} 0\le y\le a\sin\beta \\ y\cot\beta\le x\le\sqrt{a^2-y^2}\end{cases} \Leftrightarrow \begin{cases} 0\le r\sin\theta\le a\sin\beta \\ r\sin\theta\cot\beta\le r\cos\theta\le\sqrt{a^2-r^2\sin^2\theta}\end{cases} \Leftrightarrow \begin{cases} \sin\theta\ge 0,\ r\sin\theta\le a\sin\beta \\ \tan\theta\le\tan\beta,\ 0\le r\le a\end{cases} \Leftrightarrow \begin{cases} 0\le\theta\le\beta \\ 0\le r\le a\end{cases}$

$\Rightarrow \int_0^{a\sin\beta}\int_{y\cot\beta}^{\sqrt{a^2-y^2}} \ln(x^2+y^2)\,dx\,dy = \int_0^\beta\int_0^a r\ln r^2\,dr\,d\theta \overset{u=r^2}{=} \int_0^\beta\int_0^{a^2}\frac{1}{2}\ln u\,du\,d\theta$

$= \int_0^\beta\left(\frac{u}{2}\ln u\Big|_0^{a^2} - \int_0^{a^2}\frac{u}{2}\cdot\frac{1}{u}\,du\right)d\theta = \int_0^\beta\left(a^2\ln a-\frac{a^2}{2}\right)d\theta = a^2\beta\left(\ln a-\frac{1}{2}\right)$



(b) $\int_0^{a\cos\beta}\int_0^{x\tan\beta}\ln(x^2+y^2)\,dy\,dx + \int_{a\cos\beta}^{a}\int_0^{\sqrt{a^2-x^2}}\ln(x^2+y^2)\,dy\,dx$