Solutions to HW7.

§16.1

10 $\int_C (x-y+z-2)\, ds$

$= \int_0^1 [t-(1-t)+1-2]\sqrt{1^2+(-1)^2+0^2}\, dt$

$= \sqrt{2}\int_0^1 (2t-2)\, dt$

$= -\sqrt{2}$ //

12 $\int_C \sqrt{x^2+y^2}\, ds$

$= \int_{-2\pi}^{2\pi} \sqrt{(4\cos t)^2+(4\sin t)^2} \cdot \sqrt{(-4\sin t)^2+(4\cos t)^2+3^2}\, dt$

$= \int_{-2\pi}^{2\pi} 4\cdot 5\, dt = 80\pi$ //

15 $\int_C (x+\sqrt{y}-z^2)\, ds$

$= \int_0^1 (t+\sqrt{t^2}-0^2)\sqrt{1^2+(2t)^2+0^2}\, dt + \int_0^1 (1+\sqrt{1}-t^2)\sqrt{0^2+0^2+1^2}\, dt$

$= \int_0^1 2t\sqrt{1+4t^2}\, dt + \int_0^1 (2-t^2)\, dt$

$= \left[\frac{1}{6}\sqrt{1+4t^2}^{\,3}\right]_0^1 + \frac{5}{3} = \frac{5\sqrt{5}}{6} + \frac{3}{2}$ //

25 $\int_C (x+\sqrt{y})\,ds$

$$= \int_0^1 (t+\sqrt{t^2})\sqrt{1+4t^2}\,dt + \int_0^1 (1-t+\sqrt{1-t})\sqrt{(-1)^2+(-1)^2}\,dt$$

$$= \int_0^1 2t\sqrt{1+4t^2}\,dt + \sqrt{2}\int_0^1 (1-t+\sqrt{1-t})\,dt$$

$$= \frac{5\sqrt{5}}{6} - \frac{1}{6} + \sqrt{2}\left(\cancel{1}\ \frac{1}{2} + \frac{2}{3}\right)$$

$$= \frac{5\sqrt{5}}{6} - \frac{1}{6} + \cancel{\frac{5\sqrt{2}}{3}} \quad \frac{7\sqrt{2}}{6}$$

(Here we parametrize.
$y=x^2$ by (t, t^2)
$y=x$ by $(1-t, 1-t)$ $\quad t\in[0,1]$)

§16.2

5 $F = \dfrac{c}{x^2+y^2}\left(\dfrac{-x}{\sqrt{x^2+y^2}}\,i - \dfrac{y}{\sqrt{x^2+y^2}}\,j\right)$

$$= \frac{-cx}{\sqrt{x^2+y^2}^{\,3}}\,i - \frac{cy}{\sqrt{x^2+y^2}^{\,3}}\,j$$ where c is a positive constant.

6 $F = y\,i - x\,j$

7 a) $\int_{C_1} F\cdot dr = \int_0^1 (3t, 2t, 4t)\cdot(dt, dt, dt) = \int_0^1 9t\,dt = \frac{9}{2}$ //

b) $\int_{C_2} F\cdot dr = \int_0^1 (3t^2, 2t, 4t^4)\cdot(dt, 2t\,dt, 4t^3\,dt)$

$$= \int_0^1 (3t^2 + 4t^2 + 16t^7)\,dt$$

$$= \frac{7}{3} + 2 = \frac{13}{3}$$ //

7 c) $\int_{C_3 \cup C_4} F \cdot dr = \int_0^1 (3t, 2t, 0)\cdot(dt, dt, 0) + \int_0^1 (3,2,4t)\cdot(0,0,dt)$

$= \int_0^1 5t\,dt + \int_0^1 4t\,dt$

$= \frac{9}{2}$

(C_3 parametrized by $(t,t,0)$
C_4 by $(1,1,t)$ ← $t\in[0,1]$)

29 a) $\int_\gamma F_1 \cdot dr = \int_0^{2\pi} (\cos t, \sin t)\cdot(-\sin t\,dt, \cos t\,dt) = 0$

$\int_\gamma F_1 \cdot \vec{n}\,ds = \int_0^{2\pi} [\cos t\cdot\cos t - \sin t(-\sin t)]\,dt = \int_0^{2\pi} dt = 2\pi$

$\int_\gamma F_2 \cdot dr = \int_0^{2\pi} (-\sin t, \cos t)\cdot(-\sin t\,dt, \cos t\,dt) = \int_0^{2\pi} dt = 2\pi$

$\int_\gamma F_2 \cdot \vec{n}\,ds = \int_0^{2\pi} [-\sin t\cos t - \cos t(-\sin t)]\,dt = 0$

b) $\int_\gamma F_1 \cdot dr = \int_0^{2\pi} (\cos t, 4\sin t)\cdot(-\sin t\,dt, 4\cos t\,dt) = \int_0^{2\pi} 15\cos t\sin t\,dt = 0$

$\int_\gamma F_1 \cdot \vec{n}\,ds = \int_0^{2\pi} [\cos t\cdot 4\cos t - 4\sin t(-\sin t)]\,dt$

$= \int_0^{2\pi} (4\cos^2 t + 4\sin^2 t)\,dt = 8\pi$

$\int_\gamma F_2 \cdot dr = \int_0^{2\pi} (-4\sin t, \cos t)\cdot(-\sin t\,dt, 4\cos t\,dt) = \int_0^{2\pi} 4\,dt = 8\pi$

$\int_\gamma F_2 \cdot \vec{n}\,ds = \int_0^{2\pi} [-4\sin t\cdot 4\cos t - \cos t(-\sin t)]\,dt = \int_0^{2\pi} -15\cos t\sin t = 0$

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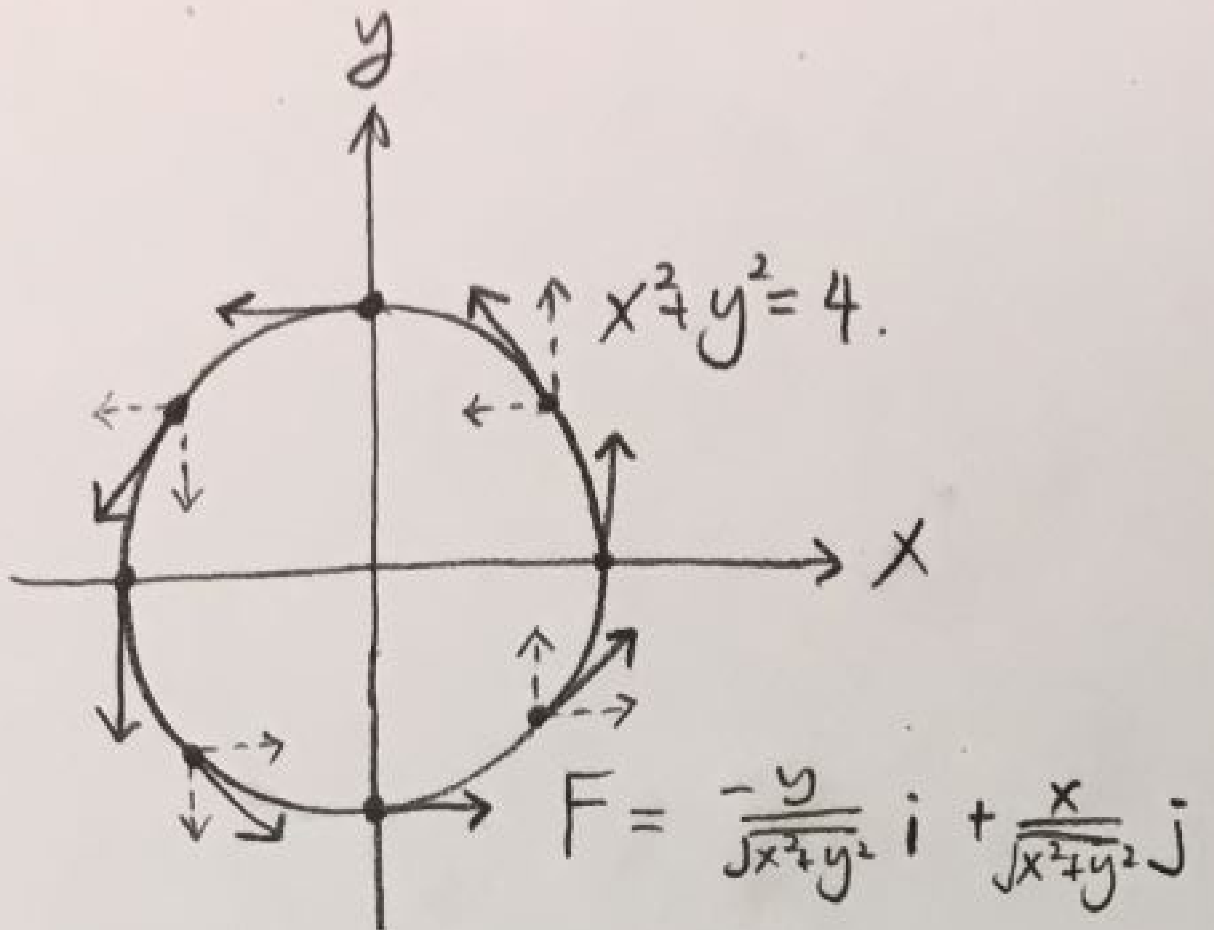

45 Yes. Work $= \int_C F \cdot dr$

$= \int_a^b (f(t), 0) \cdot (dt, f'(t)\,dt)$

$= \int_a^b f(t)\,dt$

$=$ Area.