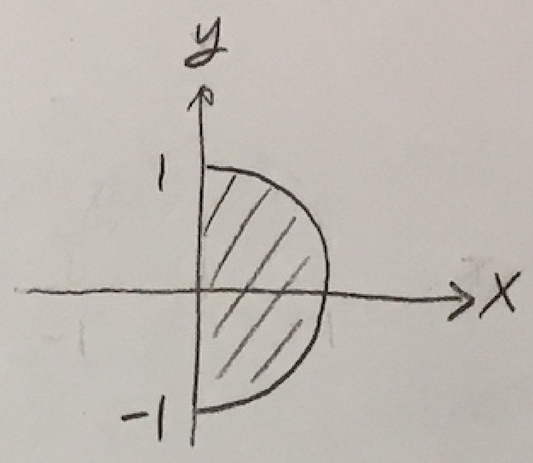
Solutions to HW4

§15.7

14 $\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{0}^{x}(x^2+y^2)\,dz\,dx\,dy$



$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{0}^{1}\int_{0}^{r\cos\theta} r^2\,dz\cdot r\,dr\,d\theta$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{0}^{1} r^3(r\cos\theta)\,dr\,d\theta$

$= \left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\theta\,d\theta\right)\left(\int_{0}^{1} r^4\,dr\right)$

$= \frac{2}{5}$. //

18 $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{\cos\theta}^{2\cos\theta}\int_{0}^{3-r\sin\theta} f(r,\theta,z)\,dz\,r\,dr\,d\theta$ //

19 $\int_{0}^{\frac{\pi}{4}}\int_{0}^{\sec\theta}\int_{0}^{2-r\sin\theta} f(r,\theta,z)\,dz\cdot r\,dr\,d\theta$ //

33 volume $= \int_{0}^{2\pi}\int_{0}^{\frac{\pi}{2}}\int_{\cos\phi}^{2} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

$= \left(\int_{0}^{2\pi}d\theta\right)\int_{0}^{\frac{\pi}{2}}\frac{1}{3}(8-\cos^3\phi)\sin\phi\,d\phi$

$= 2\pi\left[-\frac{1}{3}\left(8\cos\phi-\frac{1}{4}\cos^4\phi\right)\right]_{0}^{\frac{\pi}{2}}$

$= \frac{31}{6}\pi$ //

38 volume $= \int_0^{2\pi} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \int_0^2 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

$= \left(\int_0^{2\pi} d\theta\right)\left(\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin\phi \, d\phi\right)\left(\int_0^2 \rho^2 d\rho\right)$

$= 2\pi \times \left[-\cos\phi\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} \times \frac{8}{3}$

$= \frac{8\pi}{3}$ //

46 volume $= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_0^{-3\cos\theta} r \cdot r \, dr \, d\theta$

$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{1}{3}(-3\cos\theta)^3 \, d\theta$

$= -9 \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (1-\sin^2\theta)\cos\theta \, d\theta$

$= -9\left[\sin\theta - \frac{1}{3}\sin^3\theta\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$

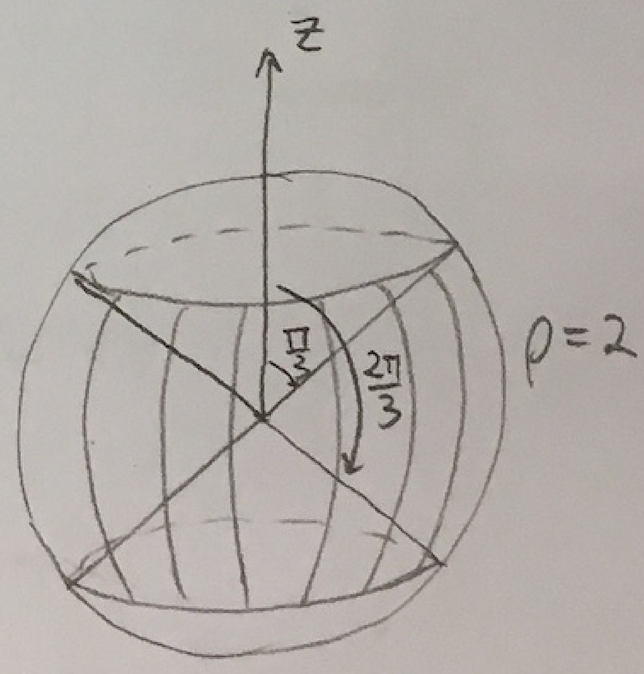
$= 12$ //

49 volume $= \int_0^{2\pi} \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \int_0^a \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

$= \left(\int_0^{2\pi} d\theta\right)\left(\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \sin\phi \, d\phi\right)\left(\int_0^a \rho^2 d\rho\right)$

$= 2\pi \times \left[-\cos\phi\right]_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \times \left(\frac{a^3}{3}\right)$

$= \frac{2\pi a^3}{3}$ //

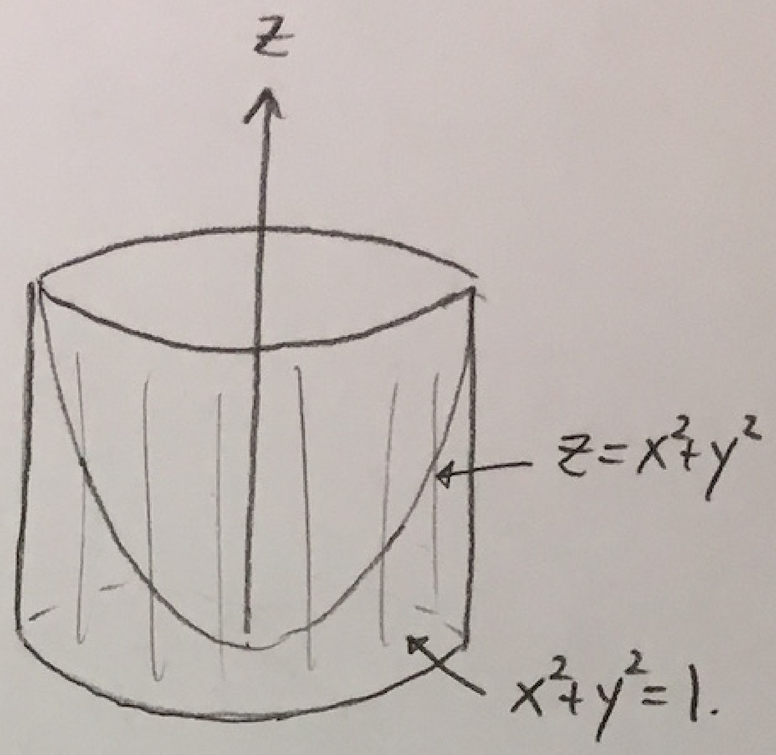


53 Volume $= \int_0^{2\pi}\int_0^1 r^2 \cdot r\,dr\,d\theta$

$= \left(\int_0^{2\pi} d\theta\right)\left(\int_0^1 r^3\,dr\right)$

$= 2\pi \times \frac{1}{4}$

$= \frac{\pi}{2}$ //



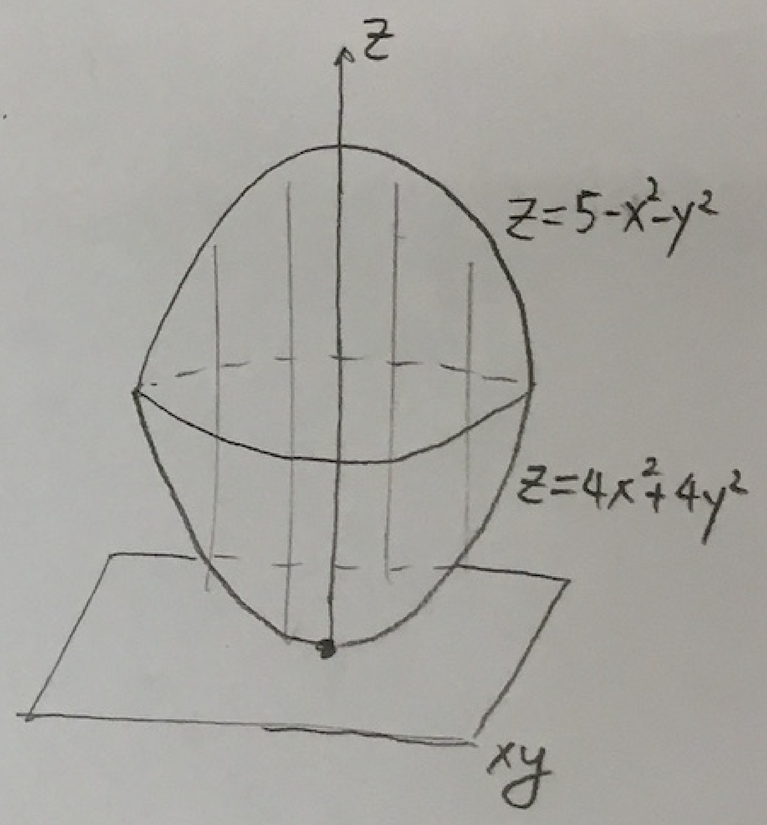
59 Notice $\{z=5-r^2\} \cap \{z=4r^2\} = \{(r,\theta,z)=(1,\theta,4) \mid \theta \in [0,2\pi]\}$

$\Rightarrow$ volume $= \int_0^{2\pi}\int_0^1 (5-r^2-4r^2)\,r\,dr\,d\theta$

$= \left(\int_0^{2\pi} d\theta\right)\left(\int_0^1 (5-5r^2)\,r\,dr\right)$

$= 2\pi \times 5 \times \left(\frac{1}{2}-\frac{1}{4}\right)$

$= \frac{5\pi}{2}$ //



64 average value $= \frac{1}{\left(\frac{4\pi}{3}\right)} \int_0^{2\pi}\int_0^{\pi}\int_0^1 (\rho\sin\phi)\cdot\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

$= \frac{3}{4\pi} \times \left(\int_0^{2\pi} d\theta\right)\left(\int_0^{\pi} \sin^2\phi\,d\phi\right)\left(\int_0^1 \rho^3\,d\rho\right)$

$= \frac{3}{4\pi} \times 2\pi \times \left[\frac{1}{2}\left(\phi-\frac{1}{2}\sin 2\phi\right)\right]_0^{\pi} \times \frac{1}{4}$

$= \frac{3\pi}{16}$ //