HW 4

§ 15.7 : 6, 8, 16, 23, 27, 36, 40, 48, 56, 63

Q6. $\int_0^{2\pi}\int_0^1\int_{-1/2}^{1/2} (r^2\sin^2\theta + z^2)\,dz\cdot r\,dr\,d\theta$

$= \int_0^{2\pi}\int_0^1 r^3\sin^2\theta + \frac{r}{12}\,dr\,d\theta$

$= \int_0^{2\pi} \frac{1}{4}\sin^2\theta + \frac{1}{24}\,d\theta$

$= \int_0^{2\pi} \frac{1}{8}(\cos 2\theta - 1) + \frac{1}{24}\,d\theta$

$= \frac{1}{8}(0 - 2\pi) + \frac{2\pi}{24}$

$= \frac{\pi}{3}$

Q8. $\int_{-1}^1\int_0^{2\pi}\int_0^{1+\cos\theta} 4r\,dr\,d\theta\,dz$

$= \int_{-1}^1\int_0^{2\pi} 2(1+\cos\theta)^2\,d\theta\,dz$

$= \int_{-1}^1\int_0^{2\pi} 2+4\cos\theta+(\cos 2\theta+1)\,d\theta\,dz$

$= \int_{-1}^1 6\pi\,dz$

$= 12\pi$

Q16. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{3\cos\theta}\int_0^{5-r\cos\theta} r\,f(r,\theta,z)\,dz\,dr\,d\theta$

Q23. $\int_0^{2\pi}\int_0^{\pi}\int_0^{(1-\cos\phi)/2} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

$= \int_0^{2\pi}\int_0^{\pi} \frac{1}{24}(1-\cos\phi)^3\sin\phi\,d\phi\,d\theta$

$= \int_0^{2\pi}\int_0^{\pi} -\frac{1}{24}(1-\cos\phi)^3\,d\cos\phi\,d\theta$

$= \int_0^{2\pi}\left[\frac{1}{96}(1-\cos\phi)^4\right]_0^{\pi} d\theta$

$= \int_0^{2\pi} \frac{1}{6}\,d\theta$

$= \frac{\pi}{3}$

Q27. $\int_0^2 \int_{-\pi}^0 \int_{\pi/4}^{\pi/2} \rho^3 \sin 2\phi \, d\phi \, d\theta \, d\rho$

$= \int_0^2 \int_{-\pi}^0 \frac{1}{2}\rho^3 \, d\theta \, d\rho$

$= \int_0^2 \frac{\pi}{2}\rho^3 \, d\rho$

$= 2\pi$

Q36. Volume $= \int_0^{2\pi} \int_0^{\pi/2} \int_0^{1-\cos\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

$= \int_0^{2\pi} \int_0^{\pi/2} \frac{1}{3}(1-\cos\phi)^3 \sin\phi \, d\phi \, d\theta$

$= \int_0^{2\pi} \frac{1}{12} \, d\theta$

$= \frac{\pi}{6}$

Q40. a) $\int_0^{\pi/2} \int_0^{3/\sqrt{2}} \int_r^{\sqrt{9-r^2}} r \, dz \, dr \, d\theta$

b) $\int_0^{\pi/2} \int_0^{\pi/4} \int_0^3 \rho^2 \sin\phi \, d\phi \, d\theta$

c) (Using b)

Volume $= \int_0^{\pi/2} \int_0^{\pi/4} \int_0^3 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

$= \int_0^3 \rho^2 \, d\rho \cdot \int_0^{\pi/2} d\theta \cdot \int_0^{\pi/4} \sin\phi \, d\phi$

$= 9 \cdot \frac{\pi}{2} \cdot \left(1 - \frac{\sqrt{2}}{2}\right)$

$= \frac{9\pi}{4}(2-\sqrt{2})$

Q48. Volume $= \int_{-\pi/2}^{\pi/2} \int_0^{\cos\theta} \int_0^{3\sqrt{1-r^2}} r \, dz \, dr \, d\theta$

$= \int_{-\pi/2}^{\pi/2} \int_0^{\cos\theta} 3r(1-r^2)^{\frac{1}{2}} \, dr \, d\theta$

$= \int_{-\pi/2}^{\pi/2} \left[-(1-r^2)^{\frac{3}{2}}\right]_0^{\cos\theta} d\theta$

$= \int_{-\pi/2}^{\pi/2} 1 - |\sin^3\theta| \, d\theta$

$= 2\int_0^{\pi/2} 1 - \sin^3\theta \, d\theta$

$= 2\left[\theta + \left(\cos\theta - \frac{1}{3}\cos^3\theta\right)\right]_0^{\pi/2}$

$= 2\left[\frac{\pi}{2} + \left(-2 + \frac{2}{3}\right)\right]$

$= \pi - \frac{4}{3}$

Q5b $\text{Volume} = \int_0^{2\pi}\int_1^{\sqrt{2}}\int_{-\sqrt{2-r^2}}^{\sqrt{2-r^2}} r\,dz\,dr\,d\theta$

$$= \int_0^{2\pi}\int_1^{\sqrt{2}} 2r(2-r^2)^{\frac{1}{2}}\,dr\,d\theta$$

$$= \int_0^{2\pi} -\frac{2}{3}(2-r^2)^{\frac{3}{2}}\Big|_1^{\sqrt{2}}\,d\theta$$

$$= \int_0^{2\pi} \frac{2}{3}\,d\theta$$

$$= \frac{4}{3}\pi$$

Q63. $\text{Average} = \frac{A}{B}$, where $A = \int_0^{2\pi}\int_0^1\int_{-1}^1 r^2\,dz\,dr\,d\theta$

$B = \pi(1)^2\cdot(2) = 2\pi$

Now, $A = \int_0^{2\pi}\int_0^1\int_{-1}^1 r^2\,dz\,dr\,d\theta$

$$= \int_0^{2\pi}\int_0^1 2r^2\,dr\,d\theta$$

$$= \int_0^{2\pi} \frac{2}{3}\,d\theta$$

$$= \frac{4\pi}{3}$$

Therefore, $\text{Average} = \frac{4\pi/3}{2\pi} = \frac{2}{3}$