MATH 3060 HW5 Due Date: Oct 24, 2016

1. Show that the only open and closed set in $\mathbb{R}$ (with standard metric $d(x,y) = |x-y|$) is the empty set and $\mathbb{R}$ itself.

2. Let $A = \{ f \in C[-1,1] : f(0) = 1, f(-1) = -1 \}$. Show that A is closed in $(C[-1,1], d_\infty)$.

3. Show that f is continuous from (X, d) to (Y, ρ) if and only if $f^{-1}(F)$ is closed in X for all closed set F in Y.

4. Identify the boundary, interior, and closure of the following sets in the indicated metric space:

(a) $[0,1] \cap \mathbb{Q}$ in ($\mathbb{R}$, standard metric)

(b) $\bigcup_{k=1}^{\infty} \left(\frac{1}{k+1}, \frac{1}{k} \right)$ in ($\mathbb{R}$, standard metric)

(c) $\mathbb{R}^2 \setminus \{ (\frac{1}{n}, 0) : n = 1,2,3,\dots \}$ in ($\mathbb{R}^2$, Euclidean metric)

(d) $\{ f \in C[0,1] : f(0) = f(1) \}$ in $(C[0,1], d_\infty)$

5. Let A and B be subset of a metric space. Show that $\overline{A \cup B} = \overline{A} \cup \overline{B}$. Does $\overline{A \cap B} = \overline{A} \cap \overline{B}$? Justify your answer.