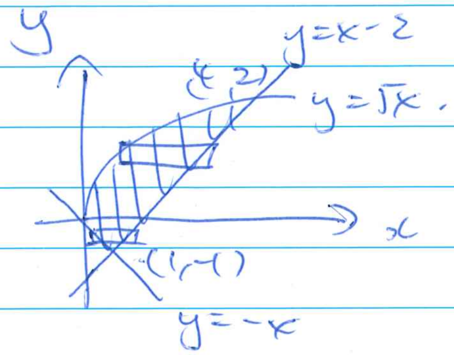
MATH 2020B HW2 solution.

15.3: 12, 18, 22

15.4: 20, 29, 42

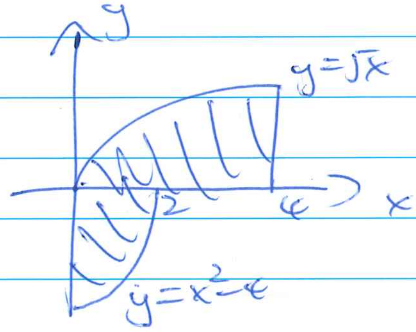
15.5: 18, 26, 32, 45.

(15.3.12).



$$\begin{aligned}\text{area} &= \int_{-1}^{0}\int_{-y}^{y+2} dx\,dy + \int_{0}^{2}\int_{y^2}^{y+2} dx\,dy \\ &= \int_{-1}^{0}(2y+2)\,dy + \int_0^2(-y^2+y+2)\,dy \\ &= \left[y^2+2y\right]_{-1}^{0} + \left[-\tfrac{1}{3}y^3+\tfrac{1}{2}y^2+2y\right]_0^2 \\ &= 1 + \tfrac{10}{3} \\ &= \tfrac{13}{3}\end{aligned}$$

(15.3.18)

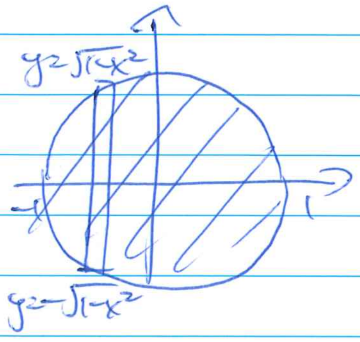


$$\begin{aligned}&\int_0^2\int_{x^2-4}^{0} dy\,dx + \int_0^4\int_0^{\sqrt{x}} dy\,dx \\ &= \int_0^2 (4-x^2)\,dx + \int_0^4 \sqrt{x}\,dx \\ &= \left[4x-\tfrac{1}{3}x^3\right]_0^2 + \left[\tfrac{2}{3}x^{3/2}\right]_0^4 \\ &= \tfrac{32}{3}.\end{aligned}$$

(15.3.22) required average

$$\begin{aligned}&= \frac{1}{(2\ln 2-\ln 2)(2\ln 2-\ln 2)}\int_{\ln 2}^{2\ln 2}\int_{\ln 2}^{2\ln 2}\frac{1}{xy}\,dx\,dy \\ &= \frac{1}{(\ln 2)^2}\int_{\ln 2}^{2\ln 2}\frac{1}{y}\left[\ln x\right]_{x=\ln 2}^{y=2\ln 2} dy \\ &= \frac{1}{(\ln 2)^2}\left[\ln(2\ln 2)-\ln\ln 2\right]\int_{\ln 2}^{2\ln 2}\frac{1}{y}\,dy \\ &= \frac{1}{(\ln 2)^2}\left[\ln(2\ln 2)-\ln\ln 2\right]^2 \\ &= \frac{1}{(\ln 2)^2}(\ln 2)^2 \quad (\because \ln(2\ln 2)=\ln 2+\ln\ln 2) \\ &= 1\end{aligned}$$

15.4.25)



$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \ln(x^2+y^2+1)\,dx\,dy$$

$$= \int_0^{2\pi}\int_0^1 \left[\ln(r^2+1)\right] r\,dr\,d\theta$$

$$= \frac{1}{2}\int_0^{2\pi}\int_{r^2+1=1}^{r^2+1=2} \ln(r^2+1)\,d(r^2+1)\,dr\,d\theta$$

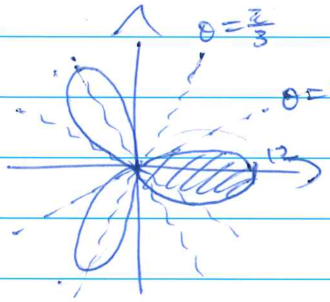
$$= \frac{1}{2}\int_0^{2\pi}\left[(r^2+1)\ln(r^2+1)-(r^2+1)\right]_{r=0}^{r=1} d\theta$$

$$= \frac{1}{2}(2\ln 2-1)\int_0^{2\pi} d\theta$$

$$= \frac{1}{2}(2\ln 2-1)(2\pi)$$

$$= \pi(2\ln 2-1)$$

15.4.29)



required area

$$= \int_{-\pi/6}^{\pi/6}\int_0^{12\cos 3\theta} r\,dr\,d\theta$$

$$= \int_{-\pi/6}^{\pi/6}\left[\frac{1}{2}r^2\right]_{r=0}^{r=12\cos 3\theta} d\theta$$

$$= \frac{1}{2}(12^2)\int_{-\pi/6}^{\pi/6}\cos^2 3\theta\,d\theta$$

$$= \frac{1}{2}(12^2)\left[\frac{1}{12}\sin 6\theta+\frac{\theta}{2}\right]_{-\pi/6}^{\pi/6}$$

$$= 12\pi$$

15.4.42) $\int_0^{\infty}\int_0^{\infty}\frac{1}{(1+x^2+y^2)^2}\,dx\,dy$

$$= \int_0^{\pi/2}\int_0^{\infty}\frac{r}{(1+r^2)^2}\,dr\,d\theta$$

$$= \int_0^{\pi/2}\left[-\frac{1}{2(1+r^2)}\right]_{r=0}^{r=\infty} d\theta$$

$$= \int_0^{\pi/2}\frac{1}{2}\,d\theta$$

$$= \pi/4$$

(15.5.18) $\int_0^1 \int_1^{\sqrt{e}} \int_1^e se^s \ln r \frac{(\ln t)^2}{t}\, dt\, dr\, ds$

$= \int_0^1 \int_1^{\sqrt{e}} se^s \ln r \left[\frac{1}{3}(\ln t)^3\right]_{t=1}^{t=e} dr\, ds$

$= \frac{1}{3}\int_0^1 \int_1^{\sqrt{e}} se^s \ln r\, dr\, ds$

$= \frac{1}{3} \int_0^1 se^s \left[r\ln r - r\right]_{r=1}^{r=\sqrt{e}} ds$

$= \frac{1}{3} (1-\sqrt{e}/2) \int_0^1 se^s\, ds.$

$= \frac{1}{3} (1-\sqrt{e}/2) \left[se^s - e^s\right]_{s=0}^{s=1}$

$= \frac{1}{3}(1-\sqrt{e}/2)(1)$

$= \frac{1}{3}(1-\sqrt{e}/2).$

(15.5.26) volume

$= \int_{-1}^1 \int_{-\sqrt{1-x^2}}^0 \int_0^{-y} dz\, dy\, dx$

$= \int_{-1}^1 \int_{-\sqrt{1-x^2}}^0 (-y)\, dy\, dx$

$= \int_{-1}^1 \left[-\frac{1}{2}y^2\right]_{y=-\sqrt{1-x^2}}^{y=0} dx$

$= \int_{-1}^1 \frac{1}{2}(1-x^2)\, dx.$

$= \left[\frac{1}{2}x - \frac{1}{6}x^3\right]_{-1}^1$

$= \frac{2}{3}$

(15.5.32) volume

$= \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{3-x} dz\, dy\, dx$

$= \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (3-x)\, dy\, dx$

$= \int_{-2}^2 2(3-x)\sqrt{4-x^2}\, dx$

$= \int_{-\pi/2}^{\pi/2} 2(3-2\sin t)(2\cos t)(2\cos t)\, dt$

$= \left[6\sin 2t + 12t + \frac{16}{3}\cos^3 t\right]_{-\pi/2}^{\pi/2}$

$= 12\pi$

(15.5.45). $\int_0^1 \int_0^{4-a-x^2} \int_a^{4-x^2-y} dz\, dy\, dx$

$= \int_0^1 \int_0^{4-a-x^2} (4-x^2-y-a)\, dy\, dx$

$= \int_0^1 \left[(4-x^2-a)^2 - \frac{1}{2}(4-a-x^2)^2 \right] dx$

$= \int_0^1 \frac{1}{2}(x^2+a-4)^2\, dx.$

$= \int_0^1 \frac{1}{2}\left[x^4 + 2(a-4)x^2 + (a-4)^2\right] dx.$

$= \frac{1}{2}\left[\frac{1}{5} + \frac{2}{3}(a-4) + (a-4)^2\right]$

$\therefore \frac{1}{2}\left[\frac{1}{5} + \frac{2}{3}(a-4) + (a-4)^2\right] = \frac{4}{15}$

$15(a-4)^2 + 10(a-4) - 5 = 0$

$3(a-4)^2 + 2(a-4) - 1 = 0$

$a-4 = -1 \text{ or } \frac{1}{3}$

$a = 3 \text{ or } \frac{13}{3}$