MATH1050 Proof-writing Exercise 8 Index of Comments

1. **DCWR.**
   You have confused 'dis-proof-by-counter-example' with 'wholesale refutation'.
   (a) To dis-prove a statement which reads

   '*There exists so-and-so belonging to the set blah-blah-blah such that so-and-so satisfies bleh-bleh-bleh*',

   you are in fact proving its negation, which reads

   '*For any so-and-so, if so-and-so belongs to the set blah-blah-blah, then so-and-so fails to satisfy bleh-bleh-bleh*'.

   In the light of this, it will not do to verify that only one specific object *so-and-so* that you are naming belongs to the set *blah-blah-blah* and fails to satisfy *bleh-bleh-bleh*.

   (b) An alternative approach for dis-proving the statement which reads

   '*There exists so-and-so belonging to the set blah-blah-blah such that so-and-so satisfies bleh-bleh-bleh*'

   is to obtain a contradiction from the assumption

   '*Suppose there existed so-and-so belonging to the set blah-blah-blah such that so-and-so satisfies bleh-bleh-bleh*'.

   In such an approach, the object *so-and-so* introduced in this assumption is something which is fixed from the moment this assumption is written. You have no control on what it is.

2. **CR.**
   You are cramming two or more ideas into the same line, thus making the whole line unclear.
   Examples.
   (a) It is unclear what is meant by

   '*Let* $u = x + y \in \mathbb{Z}$.'

   '*Let* $u = x + y$' is one thing: you are defining u to be the sum $x + y$.
   '$x + y \in \mathbb{Z}$' is another thing: you are arguing that $x + y$ is an integer because of so-and-so.

   (b) It is unclear what is meant by

   '*Suppose it were true that* $p + q \geq (s + t)^2 \geq 0$.'

   '*Suppose it were true that* $p+q \geq (s+t)^2$' is one thing: you are stating some assumption which you probably intend to use in the subsequent argument.
   '$(s + t)^2 \geq 0$' is another thing: you are arguing that $(s + t)^2$ is a non-negative real number because of so-and-so.

   (c) It is unclear what is meant by

   'There exists some $k \in \mathbb{Z}$ such that $x + z = 4 = 3k$.'

   '$x + z = 4$' is one thing: you are asserting that the sum of x, z is 4.
   '*There exists some* $k \in \mathbb{Z}$ *such that* $x+z = 3k$' is another thing: you are elaborating the statement '$x+z$ *is divisible*' according to the definition of divisibility.

   You should slow down, and write out the ideas one-by-one, expounding at most one idea within one sentence.

3. **CPL.**
   (a) The usual ordering for real numbers does not extend to complex numbers.
   For instance, it does not make sense to write something like '$z \geq |z|$' when it is not known whehter z is a real number.

   (b) When dealing with an argument concerned with complex numbers, try not to break up the complex numbers into their real and imaginary parts, unless and until it is absolutely necessary and/or very much desirable to do so.
   One good example is about dealing with equalities concerned with the modulus for complex numbers. When you use the 'relation' $|\zeta|^2 = \zeta\bar{\zeta}$, very likely you may continue playing around with multiplication of complex numbers in your subsequent calculation. Furthermore, because multiplication of complex numbers behaves nicely, your calculation is likely to remain neat and easily tractable. This is a great benefit under a modest price (of leaving possibly non-real numbers alone).
   However, when you use the 'relation' $|\zeta| = \sqrt{(\mathsf{Re}(\zeta))^2 + (\mathsf{Im}(\zeta))^2}$, you will invite the trouble of dealing with both the square root and the sum of squares simultaneously.
   Another good example (which is closely related to the above) is the use of the Triangle Inequality for complex numbers. (See Theorem (1) in the notes *Basic results on complex numbers 'beyond school mathematics'*. Also see Question (12) of Assignment 3, and Question (7) of Assignment 4.) When you break up complex numbers into their real and imaginary parts, what may follow immediately with an application the Triangle Inequality for complex numbers could be lost from sight.

4. **CTD.**

   When using the method of contradiction as one step of a multi-step argument, it is better to indicate the application of the method of contradiction and the specific statement to be justified at that step.

5. **DEF.**

   Look up the definition. You are not adhering to definition in your argument, or you have missed out key logical features in the definition. For this reason, your argument is deemed incomplete, if not altogether wrong.

6. **IA.**

   This is redundant and confusing use of the universal quantifier 'for any'.

   You have already introduced, say, a concrete object a, earlier. After that point, it is wrong to write '*for any* a *...*', because it would confuse the reader on whether you are talking about the same a, or you want to introduce another object which you quite carelessly and wrongly label as a. In fact, if you are doing things correctly, you can be (as you should be) referring to the same object a.

7. **IE.**

   This is redundant and confusing use of the existential quantifier 'there exists'/'for some'.

   You have already introduced, say, a concrete object a, earlier. After that point, it is wrong to write '*for some* a *...*', because it would confuse the reader on whether you are talking about the same a, or you want to introduce another object which you quite carelessly and wrongly label as a. In fact, if you are doing things correctly, you can be (as you should be) referring to the same object a.

8. **MA.**

   At least part of the assumptions is missing. But you are going to use these assumptions in the argument. The reader is not responsible to write out the missing assumptions for you, and will simply regard your argument as wrong when you are applying the 'missing assumptions'.

   While it is fine (and indeed nice) to state what you intend to argue for in an upcoming paragraph, when it indeed comes to that paragraph, you should make it clear what the assumptions are within the argument given in the paragraph. You must not leave it to the reader to guess what the assumption are.

   Examples:

   (a) Consider this passage below:

   > *We intend to prove the statement*
   >
   > '*For any* $z \in \mathbb{C}$, $|z + 3 - 4i| \leq |z| + 5$'
   >
   > *below:*
   >
   > *Suppose* $z \in \mathbb{C}$. *Then so-and-so-and-so-and-so. Therefore the inequality* $|z + 3 - 4i| \leq |z| + 5$ *holds.*

   The words '*For any* $z \in \mathbb{C}$' in the line

   > '*For any* $z \in \mathbb{C}$, $|z + 3 - 4i| \leq |z| + 5$'

   is part of this statement that you want to prove. When you start an argument for it, you should (still) state the assumption that you use in the argument. It is given in the words

   > '*Suppose* $z \in \mathbb{C}$'

   with which the paragraph begins.

   (b) Consider this passage below:

   > *We intend to dis-prove the statement*
   >
   > '*There exists some* $z \in \mathbb{C}$ *such that* $|z + 3 - 4i| > |z| + 5$'
   >
   > *below:*
   >
   > *Suppose there existed some* $z \in \mathbb{C}$ *such that* $|z+3-4i| > |z|+5$. *Then so-and-so-and-so-and-so. Contradiction arises.*

   The words '*There exists some* $z \in \mathbb{C}$ *such that* $|z + 3 - 4i| > |z| + 5$' give the statement that you want to dis-prove. When you start an argument against it, you should (still) state the assumption that you use in the argument. It is given in the words

   > '*Suppose there existed some* $z \in \mathbb{C}$ *such that* $|z + 3 - 4i| > |z| + 5$'

   with which the paragraph begins.

9. **MS.**

   There is a missing step which you should not have skipped.

10. **NEG.**

    Your negation for the statement concerned is wrong. Very often, it is because of a mis-interpretation of the statement to be negated.

11. **PC.**

    Punctuation and capitals should be used appropriately so as to indicate to the reader how the passage is to be read. Omissions may result in the reader being confused with the logic and/or the mathematical content in what you are writing.

12. **TR.**

    The importance of the Triangle Inequality (whether for real numbers or for complex numbers or for vectors) cannot be overstated. It is useful especially when you are dealing with inequalities involving the absolute value (for real numbers) or the modulus (for complex numbers) or the norm (for vectors).

    Make clever use of it, and you will save a lot of time and be saved from trouble.

13. **WD.**

    The deduction at this place is wrong.