MATH1050 Proof-writing Exercise 7 Index of Comments

1. **DCPC.**
   You have confused 'dis-proof-by-counter-example' with 'proof-by-contradiction'.

2. **CR.**
   You are cramming two or more ideas into the same line, thus making the whole line unclear.

   Examples.

   (a) It is unclear what is meant by

   '*Let* $u = x + y \in \mathbb{Z}$.'

   '*Let* $u = x + y$' is one thing: you are defining u to be the sum $x + y$.

   '$x + y \in \mathbb{Z}$' is another thing: you are arguing that $x + y$ is an integer because of so-and-so.

   (b) It is unclear what is meant by

   '*Suppose it were true that* $p + q \geq (s + t)^2 \geq 0$.'

   '*Suppose it were true that* $p+q \geq (s+t)^2$' is one thing: you are stating some assumption which you probably intend to use in the subsequent argument.

   '$(s + t)^2 \geq 0$' is another thing: you are arguing that $(s + t)^2$ is a non-negative real number because of so-and-so.

   (c) It is unclear what is meant by

   'There exists some $k \in \mathbb{Z}$ such that $x + z = 4 = 3k$.'

   '$x + z = 4$' is one thing: you are asserting that the sum of x, z is 4.

   '*There exists some* $k \in \mathbb{Z}$ *such that* $x+z = 3k$' is another thing: you are elaborating the statement '$x+z$ *is divisible*' according to the definition of divisibility.

   You should slow down, and write out the ideas one-by-one, expounding at most one idea within one sentence.

3. **DEF.**
   Look up the definition. You are not adhering to definition in your argument, or you have missed out key logical features in the definition. For this reason, your argument is deemed incomplete, if not altogether wrong.

4. **EADC.**
   When giving a dis-proof by counter-example, you are in fact proving a statement starting with the existential quantifier, which reads: '*There exists some blah-blah-blah such that bleh-and-bleh*'.

   A correct argument should start with the words '*Take so-and-so*', through which you specify to the readers the objects, namely so-and-so, which you expect will serve as a counter-example against the statement to be dis-proved. Next you should proceed to verify that this specific so-and-so is indeed a counter-example against the statement to be dis-proved.

   For detail, read the examples in the Handout *Dis-proofs*.

5. **IA.**
   This is redundant and confusing use of the universal quantifier 'for any'.

   You have already introduced, say, a concrete object a, earlier. After that point, it is wrong to write '*for any* a *...*', because it would confuse the reader on whether you are talking about the same a, or you want to introduce another object which you quite carelessly and wrongly label as a. In fact, if you are doing things correctly, you can be (as you should be) referring to the same object a.

6. **IE.**
   This is redundant and confusing use of the existential quantifier 'there exists'/'for some'.

   You have already introduced, say, a concrete object a, earlier. After that point, it is wrong to write '*for some* a *...*', because it would confuse the reader on whether you are talking about the same a, or you want to introduce another object which you quite carelessly and wrongly label as a. In fact, if you are doing things correctly, you can be (as you should be) referring to the same object a.

7. **IL.**
   Find out the laws of indices.

8. **MA.**
   At least part of the assumptions is missing. But you are going to use these assumptions in the argument. The reader is not responsible to write out the missing assumptions for you, and will simply regard your argument as wrong when you are applying the 'missing assumptions'.

9. **MS.**
   There is a missing step which you should not have skipped.

10. **NEG.**

Your negation for the statement concerned is wrong. Very often, it is because of a mis-interpretation of the statement to be negated.

Example. Consider the statement (M):

> (M): '*Let* $x, y, z \in \mathbb{N}$. *Suppose* $x + y$ *is divisible by* 3 *and* $y + z$ *is divisible by* 3. *Then* $x + z$ *is divisible by* 5.'

Expressed in the most formal way, with all the quantifiers explicitly presented, (M) reads:

> (M): '*For any* $x, y, z \in \mathbb{N}$, *if* $x + y$ *is divisible by* 3 *and* $y + z$ *is divisible by* 3 *then* $x + z$ *is divisible by* 5.'

So the negation $(\sim M)$ of (M) is:

> $(\sim M)$: '*There exist some* $x, y, z \in \mathbb{N}$ *such that* $x + y$ *is divisible by* 3 *and* $y + z$ *is divisible by* 3 *and* $x + z$ *is not divisible by* 5.'

To dis-prove (M) is to prove $(\sim M)$.

Carefully read the notes on *Universal quantifier and existential quantifier*, *Statements with several quantifiers*, *Disproofs*.

11. **PC.**

Punctuation and capitals should be used appropriately so as to indicate to the reader how the passage is to be read. Omissions may result in the reader being confused with the logic and/or the mathematical content in what you are writing.

12. **RC.**

The symbols ' $\sqrt[n]{}$ ' should be used judiciously (if not avoided at all) when it comes to complex numbers.

In school maths, when we are given a non-negative real number r and an integer greater than 1, we are used to denoting by $\sqrt[n]{r}$ the one and only one non-negative real number whose n-th power is number r. What we are doing is that we privilege the and only one of the n-th roots of r in the complex number because of its being real and non-negative. For instance:

- When we write $\sqrt{4}$, we mean 2: we are privileging 2 over -2 because the former is non-negative.

  When we $\sqrt[4]{81}$, we mean 3: we are privileging 3 over -3, $3i$, $-3i$ because 3 is non-negative.

The notation fails to generalize for the n-th roots of complex numbers which are not necessarily real and positive.

The reason is that when α is a non-real-and-non-negative complex number and n is an integer greater than 1, there are altogether n pairwise distinct n-th roots of α in the complex numbers, amongst which none should be privileged over any one else. For this reason, it does not make sense to write ' $\sqrt[n]{\alpha}$'.

For instance:

- What privileges $2i$ over $-2i$ (or the other way round) as square roots of -4? (So it makes no sense to write something like ' $\sqrt{2i}$ '.)

  What privileges one of $1+i$, $1-i$, $-1+i$, $-1-i$ over the others as quartic roots of -4? (So it makes no sense to write something like ' $\sqrt[4]{2i}$ '.)

  What privileges one of $\frac{\sqrt{3}}{2} + \frac{1}{2}i$, $-\frac{\sqrt{3}}{2} + \frac{1}{2}i$, $-i$ as cubic roots of i? (So it makes no sense to write something like ' $\sqrt[3]{i}$ '.)

For more detail on the notion of n-th roots in complex numbers, refer to the Handouts *Basic results on complex numbers 'beyond school mathematics'*, *De Moivre's Theorem and roots of unity*.

13. **VD.**

While Venn diagram helps us in doing exploration, it cannot substitute the formal argument (whether in the context of a proof or that of a dis-proof).

14. **WD.**

The deduction at this place is wrong.