

## MATH1050 Proof-writing Exercise 6

### Advice.

- Study the Handouts *Examples of proofs concerned with ‘subset relations’*, *Examples of proofs for properties of basic set operations* before answering the questions.
- Besides the handouts mentioned above, Questions (9), (12a) of Exercise 6 may also be relevant to some of the questions below.
- When giving an argument, remember to adhere to definition, always. And don't forget to make the argument self-contained: the assumptions needed in the argument should be stated, (likely) at the the top.

- i. Explain the phrase *subset of a set* by giving its appropriate definition.
    - ii. Explain the phrase *being not a subset of a set*, in terms of the notion of *belonging*, by giving an appropriate mathematical statement (whose content is self-contained).
  - (b) Let  $A = \{\zeta \in \mathbb{C} : |\zeta - i| < 1\}$ ,  $B = \{\zeta \in \mathbb{C} : |\zeta + i| < 3\}$ .
    - i. Is it true that  $A \subset B$ ? Justify your answer.
    - ii. Is it true that  $B \subset A$ ? Justify your answer.
- (a) Explain the phrase *union of two sets* by stating its appropriate definition.
  - (b) Prove statement below, with reference to the definitions of set equality, subset relation, intersection, union, complement, where appropriate.
    - Let  $A, B, C, D$  be sets. Suppose  $A \subset C$  and  $B \subset D$ . Then  $A \cup B \subset C \cup D$ .
- (a) Explain the phrases *intersection of two sets*, *complement of a set in another (not distinct) set* by stating their appropriate definitions.
  - (b) Prove the statements below, with reference to the definitions of set equality, subset relation, intersection, union, complement, where appropriate.
    - i. Let  $A, B$  be sets. Suppose  $A \subset A \setminus B$ . Then  $A \cap B = \emptyset$ .
    - ii. Let  $A, B$  be sets. Suppose  $A \cap B = \emptyset$ . Then  $A \subset A \setminus B$ .
4. We introduce/recall the definition of *column space* for matrices (with real entries):

Let  $A$  be an  $(m \times n)$ -matrix with real entries.

The column space of  $A$  is defined to be the set  $\left\{ \mathbf{y} \in \mathbb{R}^m : \begin{array}{l} \text{There exists some } \mathbf{x} \in \mathbb{R}^n \\ \text{such that } \mathbf{y} = A\mathbf{x} \end{array} \right\}$ .

It is denoted by  $\mathcal{C}(A)$ .

Prove the statements below, with reference to the definition of *column space*:

- (a) Suppose  $B$  is an  $(n \times n)$ -square matrix with real entries. Then  $\mathcal{C}(B^2) \subset \mathcal{C}(B)$ .
- (b) Let  $D$  be an  $(m \times n)$ -matrix with real entries, and  $E, F$  be  $(n \times p)$ -matrices with real entries. Suppose  $\mathcal{C}(E) \subset \mathcal{C}(F)$ . Then  $\mathcal{C}(DE) \subset \mathcal{C}(DF)$ .