MATH1050 Proof-writing Exercise 5 Index of Comments

1. **LFMI.**
   (a) The 'main body' in an application of mathematical induction on a proposition, say, $P(n)$, for proving '*$P(n)$ is true for every $n \in \mathbb{N}$*', is expected to be made of two logically independent passages, known as the 'initial step argument' and the 'induction argument'.
   The purpose of the 'initial step argument' is to verify the statement
   ($\star$) '*$P(0)$ is true.*'
   The purpose of the 'inductive argument' is to verify the statement
   ($\star\star$) '*for any $k \in \mathbb{N}$, if $P(k)$ is true then $P(k+1)$ is true.*'
   With the completion of the 'initial step argument' and the 'inductive argument', the specific '$P(n)$' under question is assured to have satisfied the assumption in the statement known as the 'Principle of Mathematical Induction'. In the light of this, the conclusion in the 'Principle of Mathematical Induction' may be invoked on this specific $P(n)$:
   '*$P(n)$ is true for any $n \in \mathbb{N}$.*'
   (b) The 'initial step argument' should start with the stating of the assumption in the statement $P(0)$.
   The main body of the 'initial step argument' should be about deducing the conclusion in the statement $P(0)$ from the assumption in the statement $P(0)$.
   It is wrong to write, at the end of the 'initial step argument', something like
   '*Hence $n = 0$ is true.*'
   (c) The 'inductive argument' should start with the words:
   '*Let k be blah-blah-blah. Suppose $P(k)$ is true.*'
   which is usually referred to as the 'inductive hypothesis'.
   The 'inductive argument' should close with the words
   '*$P(k+1)$ is true.*'
   The main body of the 'inductive argument' should be about deducing the statement $P(k+1)$, probably with help provided from the statement $P(k)$, and on its own should begin with the stating of the assumption in the statement $P(k+1)$, and 'moves' towards the conclusion in the statement $P(k+1)$.
   It is wrong to write, in place of the 'inductive hypothesis', anything of the likes of
   '*Suppose $n = k$ is true.*'
   or
   '*Suppose $n = k$.*'
   It is wrong to write, at the end of the 'inductive argument', something like
   '*Hence $n = k+1$ is true.*'

2. **PROP.**
   (a) Find out what exactly is the proposition, say, $P(n)$, upon which mathematical induction is going to be applied.
   State it explicitly before you proceed to work out the 'main body' in the mathematical induction.
   Depending on the nature of the statement to be proved, it can happen that some 'over-arching assumption' needs to be stated prior to the formulation of the specific '$P(n)$' under question.
   Ill-formulated propositions will screw up everything.
   (b) For instance, when you want to apply mathematical induction to prove the statement
   '*Let $n \in \mathbb{N}\backslash\{0,1\}$, and r be a real number greater than 1. Suppose $a_1, a_2, \cdots, a_n$ are positive real numbers. Then* $\log_r\left(\prod_{j=1}^{n} a_j\right) = \sum_{j=1}^{n} \log_r(a_j)$.'
   The statement '*r is (assumed to be) a real number greater than* 1' should serve as an 'overarching assumption' that you need to state prior to the formulation of $P(n)$.
   Put in plain words, the '$P(n)$' under question should be saying something like '*for any (choice of) n positive real numbers, the equality blah-blah-blah holds for these n positive real numbers*'.
   For this reason, an appropriate formulation of $P(n)$ is:
   '*Suppose $a_1, a_2, \cdots, a_n$ are positive real numbers. Then* $\log_r\left(\prod_{j=1}^{n} a_j\right) = \sum_{j=1}^{n} \log_r(a_j)$.'
   It is wrong to formulate $P(n)$ as:
   '$\log_r\left(\prod_{j=1}^{n} a_j\right) = \sum_{j=1}^{n} \log_r(a_j)$'
   (This wrong formulation makes no sense because of the ambiguity on the a_j's.)