MATH1050 Proof-writing Exercise 1 (Answers and selected solution)

1. **Answer.**
   Let u, v be integers. The integer u is said to be divisible by the integer v if there exists some integer k such that $u = kv$.

2. **Solution.**
   (a) $0 = 0 \cdot 0$.
   0 is an integer.
   Therefore 0 is divisible by 0.
   (b) Let x be an integer. Suppose x is divisible by 0.
   Then, by definition, there exists some integer k such that $x = k \cdot 0$.
   Therefore (for the same k), we have $x = k \cdot 0 = 0$.

3. **Solution.**
   Let $x, y \in \mathbb{Z}$. Suppose x is divisible by y and y is divisible by x.
   Since x is divisible by y, there exists some $k \in \mathbb{Z}$ such that $x = ky$.
   Since y is divisible by x, there exists some $\ell \in \mathbb{Z}$ such that $y = \ell x$.
   We have $x = ky = k\ell x$.
   Then $(k\ell - 1)x = 0$.
   Therefore $x = 0$ or $k\ell = 1$.
   - (Case 1). Suppose $x = 0$. Then $y = 0$. Therefore $|x| = |y|$.
   - (Case 2). Suppose $x \neq 0$. Then $1 = k\ell$. Since k, ℓ are integers, we have $k = \ell = 1$ or $k = \ell = -1$. Then $|x| = |ky| = |y|$.

   Hence in any case, $|x| = |y|$.