MATH1050 Workshop on Proof-writing 2

1. Let $A = \{x \mid x = r^6 \text{ for some } r \in \mathbb{Q}\}$, $B = \{x \mid x = r^2 \text{ for some } r \in \mathbb{Q}\}$.
   (a) Prove that $A \subset B$.
   (b) Prove that $B \not\subset A$.
2. Prove the statement ($\star$) below:
   ($\star$) *Let A, B, C be sets. Suppose $A \subset B$, $B \subset C$, and $C \subset A$. Then $A = B$.*
3. Prove the statement ($\star$) below:
   ($\star$) *Let A, B be sets. Suppose $A \subset A \backslash B$. Then $A \cap B = \emptyset$.*
4. Dis-prove the statement ($\star$) below:
   ($\star$) *Suppose A, B, C are non-empty sets. Then $B \backslash A \subset (C \backslash A) \backslash (C \backslash B)$.*
5. (a) Prove the statement ($\sharp$) below:
   ($\sharp$) *Let A, B be sets, and $f : A \longrightarrow B$ be a function. For any subset S of A, $S \subset f^{-1}(f(S))$.*
   (b) Dis-prove the statement ($\flat$) below:
   ($\flat$) *Let A, B be sets, and $f : A \longrightarrow B$ be a function. For any subset S of A, $f^{-1}(f(S)) \subset S$.*