# MATH1050 Workshop on Proof-writing 1

1. Prove the statement ($\star$) below, with reference to the definition for *divisibility*:

   ($\star$) *Let $x, n \in \mathbb{Z}$. Suppose $x$ is divisible by $n$. Then for any $y \in \mathbb{Z}$, $(5x+y)^7 + (5x-y)^7$ is divisible by $10n$.*

2. Prove the statement ($\star$) below, with reference to the definition for *rational numbers*:

   ($\star$) *Let $x, y$ be rational numbers. Suppose $y \neq 0$. Then $\dfrac{x}{y}$ is a rational number.*

3. Prove the statement ($\star$) below, with reference to the definitions for *divisibility* and *prime number*:

   ($\star$) *Let $p, q$ be prime numbers. Suppose $p, q$ are positive and $p \neq q$. Then $p$ is not divisible by $q$.*

4. Prove the statement ($\star$) below, with reference to the definition of *strict monotonicity*:

   ($\star$) *Let $a$ be a real number, and $h : (0, +\infty) \longrightarrow \mathbb{R}$ be the real-valued function of one real variable defined by $h(x) = x + \dfrac{a^2}{x}$ for any $x \in (0, +\infty)$. Suppose $a > 0$. Then $h$ is strictly decreasing on $(0, a]$.*

5. Prove the statement ($\star$) below:

   ($\star$) *Let $a, b$ be real numbers. Suppose $|a| \leq 1$ and $|b| \leq 1$. Then $\sqrt{1-a^2} + \sqrt{1-b^2} \leq 2\sqrt{1-(a+b)^2/4}$.*

6. Prove the statement ($\star$) below:

   ($\star$) *Let $m, n \in \mathbb{N} \backslash \{0\}$. Let $x$ be a positive real number. Suppose $m > n$. Then $x^m + \dfrac{1}{x^m} \geq x^n + \dfrac{1}{x^n}$. Moreover, equality holds iff $x = 1$.*