MATH1050 Exercise 6 Supplement (Answers)

1. (a) Let P, Q be statements.

   $[(P \to Q) \wedge P] \to Q$ is true irrespective of the truth values of P, Q. Hence it is a tautology.

   This is a truth table displaying the truth values of P, Q, ..., $[(P \to Q) \wedge P] \to Q$:

P	Q	$P \to Q$	$(P \to Q) \wedge P$	$[(P \to Q) \wedge P] \to Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

   Remark. For this reason, $[(P \to Q) \wedge P] \to Q$ is a rule of inference. We often use it in arguments, for example:

   - If it is Sunday, John goes hiking. It is Sunday. Therefore John goes hiking.

   (b) Let P, Q be statements. $[(P \to Q) \wedge Q] \to P$ is a contingent statement. It is true when P, Q are both true. It is false when P is false and Q is true.

   This is the truth table displaying the truth values of P, Q, ..., $[(P \to Q) \wedge Q] \to P$:

P	Q	$P \to Q$	$(P \to Q) \wedge Q$	$[(P \to Q) \wedge Q] \to P$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

   Remark. Is there something wrong in the following argument?

   - If it is Sunday, John goes hiking. John goes hiking. Therefore it is Sunday.

2. Let P, Q be statements. The statements $P \leftrightarrow (\sim Q)$, $(\sim P) \leftrightarrow Q$, $(P \vee Q) \wedge [\sim(P \wedge Q)]$, $[P \wedge (\sim Q)] \vee [(\sim P) \wedge Q]$ are logically equivalent to each other.

   Here are two truth tables which display the truth values of P, Q and the statements $P \leftrightarrow (\sim Q)$, $(\sim P) \leftrightarrow Q$, $(P \vee Q) \wedge [\sim(P \wedge Q)]$, $[P \wedge (\sim Q)] \vee [(\sim P) \wedge Q]$:

P	Q	$\sim P$	$\sim Q$	$P \leftrightarrow (\sim Q)$	$(\sim P) \leftrightarrow Q$	$P \vee Q$	$P \wedge Q$	$\sim(P \wedge Q)$	$(P \vee Q) \wedge [\sim(P \wedge Q)]$
T	T	F	F	F	F	T	T	F	F
T	F	F	T	T	T	T	F	T	T
F	T	T	F	T	T	T	F	T	T
F	F	T	T	F	F	F	F	T	F

P	Q	$\sim P$	$\sim Q$	$P \wedge (\sim Q)$	$(\sim P) \wedge Q$	$[P \wedge (\sim Q)] \vee [(\sim P) \wedge Q]$	$P \leftrightarrow (\sim Q)$
T	T	F	F	F	F	F	F
T	F	F	T	T	F	T	T
F	T	T	F	F	T	T	T
F	F	T	T	F	F	F	F

3. (a) The statements $P \to (Q \wedge R)$, $(P \to Q) \wedge (P \to R)$ are logically equivalent.

P	Q	R	$Q \wedge R$	$P \to (Q \wedge R)$	$P \to Q$	$P \to R$	$(P \to Q) \wedge (P \to R)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	F	F	T	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

   Remark. This logical equivalence is used so frequently that we are not even aware of it when using it. For intance, compare the statements (A), (B) below:

   (A) Let $n \in \mathbb{Z}$. Suppose n is divisible by 6. Then (n is divisible by 2 and n is divisible by 3).

   (B) Let $n \in \mathbb{Z}$. The following statements hold:

   - Suppose n is divisible by 6. Then n is divisible by 2.
   - Suppose n is divisible by 6. Then n is divisible by 3.

   (b) The statements $P \to (Q \to R)$, $(P \wedge Q) \to R$ are logically equivalent.

P	Q	R	$Q \to R$	$P \to (Q \to R)$	$P \wedge Q$	$(P \wedge Q) \to R$
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	T	T	F	T
T	F	F	T	T	F	T
F	T	T	T	T	F	T
F	T	F	F	T	F	T
F	F	T	T	T	F	T
F	F	F	T	T	F	T

Remark. This logical equivalence is also used so frequently that we are not even aware of it when using it. For intance, compare the statements (A), (B) below:

(A) Let $n \in \mathbb{Z}$. Suppose n is divisible by 2. Further suppose n is divisible by 3. Then n is divisible by 6.

(B) Let $n \in \mathbb{Z}$. Suppose (n is divisible by 2 and n is divisible by 3). Then n is divisible by 6.

(c) The statements $P \to (Q \vee R)$, $(P \to Q) \vee (P \to R)$ are logically equivalent.

P	Q	R	$Q \vee R$	$P \to (Q \vee R)$	$P \to Q$	$P \to R$	$(P \to Q) \vee (P \to R)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T

(d) The statements $(P \vee Q) \to R$, $(P \to R) \wedge (Q \to R)$ are logically equivalent.

P	Q	R	$P \vee Q$	$(P \vee Q) \to R$	$P \to R$	$Q \to R$	$(P \to R) \wedge (Q \to R)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

Remark. To prove a statement of the form 'if (blah-blah-blah or bleh-bleh-bleh) then blih-blih-blih', we can prove the statements 'if blah-blah-blah then blih-blih-blih', 'if bleh-bleh-bleh then blih-blih-blih' separately.

(e) The statements $(P \to Q) \to R$, $P \to (Q \to R)$ are not logically equivalent.

P	Q	R	$P \to Q$	$(P \to Q) \to R$	$Q \to R$	$P \to (Q \to R)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	F	T	T

Remark. In general, the statements (†), (‡) are distinct:

(†) Suppose that if blah-blah-blah then bleh-bleh-bleh. Then blih-blih-blih.

(‡) Suppose blah-blah-blah. Then, if bleh-bleh-bleh then blih-blih-blih.

(f) The statements $P \to (Q \vee R)$, $[P \wedge (\sim Q)] \to R$ are logically equivalent.

P	Q	R	$Q \vee R$	$P \to (Q \wedge R)$	$\sim Q$	$P \wedge (\sim Q)$	$[P \wedge (\sim Q)] \to R$
T	T	T	T	T	F	F	T
T	T	F	T	T	F	F	T
T	F	T	T	T	T	T	T
T	F	F	F	F	T	T	F
F	T	T	T	T	F	F	T
F	T	F	T	T	F	F	T
F	F	T	T	T	T	F	T
F	F	F	F	T	T	F	T

4. (a) The statement $[P \to (P \to Q)] \to (P \to Q)$ is a tautology.

P	Q	$P \to Q$	$P \to (P \to Q)$	$[P \to (P \to Q)] \to (P \to Q)$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

.

Remark. When we try to arrive at the conclusion Q from the assumption P, we are allowed to apply P more than once in our argument. We can apply it twice, thrice ... and the deduction is not falsified.

(b) The statement $(P \to R) \to [(P \wedge Q) \to R)]$ is a tautology.

P	Q	R	$P \wedge R$	$P \to Q$	$(P \wedge R) \to Q$	$(P \to R) \to [(P \wedge Q) \to R)]$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	F	T	T	T
T	F	F	F	F	T	T
F	T	T	F	T	T	T
F	T	F	F	T	T	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T

Remark. When the conclusion Q can be deduced from the assumption P, the conclusion Q can also be deduced from the assumptions P and R.

(c) The statement $[(P \to Q) \wedge (Q \to R)] \to (P \to R)$ is a tautology.

P	Q	R	$P \to Q$	$Q \to R$	$P \to R$	$(P \to Q) \wedge (Q \to R)$	$[(P \to Q) \wedge (Q \to R)] \to (P \to R)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Remark. This is known as hypothetical syllogism. From 'if it is Saturday, John watches football' and 'if John watches football, John goes to bed late', we deduce 'if it is Saturday, John goes to bed late'.

(d) The statement $[(P \to Q) \wedge (Q \to R) \wedge (R \to P)] \to (Q \to P)$ is a tautology.
Write $P \to Q$ as S, $Q \to R$ as T, $R \to P$ as U, and $Q \to P$ as V

P	Q	R	S	T	U	V	$S \wedge T \wedge U$	$(S \wedge T \wedge U) \to V$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	F	T
T	F	T	F	T	T	T	F	T
T	F	F	F	T	T	T	F	T
F	T	T	T	T	F	F	F	T
F	T	F	T	F	T	F	F	T
F	F	T	T	T	F	T	F	T
F	F	F	T	T	T	T	T	T

(e) The statement $(P \to R) \to [(P \to Q) \vee (Q \to R)]$ is a tautology.

P	Q	R	$P \to Q$	$Q \to R$	$P \to R$	$(P \to Q) \vee (Q \to R)$	$(P \to R) \to [(P \to Q) \vee (Q \to R)]$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	F	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

(f) The statement $(P \to Q) \to [(Q \to R) \vee (P \wedge R)]$ is neither a tautology nor a contradiction; it is a contingent statement.

P	Q	R	$P \to Q$	$Q \to R$	$P \wedge R$	$(Q \to R) \vee (P \wedge R)$	$(P \to Q) \to [(Q \to R) \vee (P \wedge R)]$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	T	T	T
T	F	F	F	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	T	F	F	F	F
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

5. (a) $0, 1, 2, 3$.
   (b) $0, 1, 2, 3, 4$.
   (c) $0, 1, 2, \{0\}, \{1\}$.
   (d) $0, 1, \{0, 1\}$.
   (e) $0, 1, \{0, 1\}, \{\{0, 1\}\}$.
6. (a) $0, 1, 2, 3$.
   (b) $0, 1, 2$.
   (c) $0, 1, 3$.
   (d) $0, 2$.
   (e) $0, 1, 2, 3, \{1\}, \{2, 3\}$.
   (f) $0, 1, 2, 3, \{1, 2\}, \{3\}$.
   (g) $1, 2, 3, \{1, 2\}, \{3\}$.
   (h) 2.
   (i) $\{2, 3\}$.

(j) $\{2\}, \{3\}$.

(k) $\{1, 2\}$.

(l) $\emptyset$, $\{0\}$, $\{2\}$, $\{\{1\}\}$, $\{0,2\}$, $\{0,\{1\}\}$, $\{2,\{1\}\}$, $\{0,2,\{1\}\}$.

7. (a) $0, 1, 2, 3, 4$.
   (b) $0, 1, \{1,2,3\}, \{\{3\},4\}$.
   (c) $0, 1$.
   (d) $0, 1, 2, 3, 4, \{1,2,3\}, \{\{3\},4\}$.
   (e) $0, 1, 2$.
   (f) $\{1,2,3\}$, $\{\{3\},4\}$.
   (g) $0, 1, 2, \{1,2,3\}$, $\{\{3\},4\}$.
   (h) $\emptyset$, $\{0\}$, $\{1\}$, $\{0,1\}$.
8. (a) $\{3,5\}$.
   (b) $\{3,5\}, \{5,7\}, 1, \{5\}$.
   (c) $\{5,7\}$.
   (d) $\emptyset, \{\{3,5\}\}, \{\{5,7\}\}, \{\{3,5\},\{5,7\}\}$.
9. (a) $\{h\}, \{n\}$.
   (b) $\{b,e\}, \{e\}, \{t\}, \{h\}, \{o,v\}, \{n\}, \{a,y,d\}$.
   (c) $\{e\}, \{o,v\}, \{n\}$.
   (d) $\emptyset, \{\{m,o\}\}, \{\{z,a,r,t\}\}, \{\{m,o\},\{z,a,r,t\}\}$.
10. (a) TEN.
    (b) TWO.
    (c) ONE.
    (d) a, t.
    (e) $\emptyset$, $\{a\}$, $\{t\}$, $\{a,t\}$.
11. (a) Seven
    (b) Nine
    (c) Four
    (d) Seven
    (e) Two
    (f) One
    (g) s, u
    (h) $\emptyset, \{s\}, \{u\}, \{s,u\}$
12. (a) Ten.
    (b) One.
    (c) Four.
    (d) One.
    (e) u, o, d.
    (f) $\emptyset, \{u\}, \{o\}, \{d\}, \{u,o\}, \{o,d\}, \{u,d\}, \{u,o,d\}$.
13. (a) Two
    (b) Four
    (c) Three
    (d) $\emptyset$
    (e) $\mathbb{N}$
    (f) $\{\{\emptyset\}\}$, $\{\mathbb{N}\}$, G.
14. (a) $A = \emptyset$.
    (b) $B \neq \emptyset$; 8 is an element of B.
15. (a) $A = \emptyset$.
    (b) $B \neq \emptyset$; 8 is an element of B.
16. (a) True. (b) False. (c) False. (d) True. (e) False. (f) True.

17. (a) $\left\{x \in \mathbb{R} : x = \dfrac{1}{2^n} \text{ for some } n \in \mathbb{N}\right\}$.

    Alternative answers: $\left\{x \in \mathbb{Q} : x = \dfrac{1}{2^n} \text{ for some } n \in \mathbb{N}\right\}$, $\left\{x \in \mathbb{C} : x = \dfrac{1}{2^n} \text{ for some } n \in \mathbb{N}\right\}$,
    $\left\{x \,\middle|\, x = \dfrac{1}{2^n} \text{ for some } n \in \mathbb{N}\right\}$, ...

    Also acceptable: $\left\{\dfrac{1}{2^n} \,\middle|\, n \in \mathbb{N}\right\}$, ...

    Remark. In the expression '$\left\{x \in \mathbb{R} : x = \dfrac{1}{2^n} \text{ for some } n \in \mathbb{N}\right\}$', the left-hand-side of the colon tells us from which 'larger set' $\mathbb{R}$ the elements of the constructed set are to be taken, while the right-hand-side of the colon tells us the 'condition' which the x's to be 'put inside' the constructed set has to satisfy exactly. This 'condition' is a 'predicate with variable x', so that whenever x is fixed it becomes a (mathematical) statement for which it makes sense to tell whether it is true or false.

    It is wrong to write this expression as '$\left\{x = \dfrac{1}{2^n} \text{ for some } n \in \mathbb{N} : x \in \mathbb{R}\right\}$'.

    Each of the following is also wrong:
    $\left\{x \mid x = \dfrac{1}{2^n} \text{ for any } n \in \mathbb{N}\right\}$, $\left\{x \mid x = \dfrac{1}{2^n} \text{ where } n \in \mathbb{N}\right\}$, $\left\{x \mid x = \dfrac{1}{2^n} \text{ and } n \in \mathbb{N}\right\}$, $\left\{x \mid x = \dfrac{1}{2^n}, n \in \mathbb{N}\right\}$.

    (b) $\left\{x \in \mathbb{R} : x = \dfrac{3^n}{5^n} \text{ for some } n \in \mathbb{N}\right\}$.

    (c) $\left\{x \in \mathbb{R} : x = \dfrac{2^m}{3^n} \text{ for some } m, n \in \mathbb{N}\right\}$.

    Alternative answers: $\left\{x \in \mathbb{Q} : x = \dfrac{2^m}{3^n} \text{ for some } m, n \in \mathbb{N}\right\}$, $\left\{\dfrac{2^m}{3^n} \,\middle|\, m \in \mathbb{N} \text{ and } n \in \mathbb{N}\right\}$, ...

    (d) $\{x \in \mathbb{R} : x = \pi^m - e^n \text{ for some } m, n \in \mathbb{N}\}$.

18. (a) The general solution of the equation is given by $x = \frac{\pi}{2} + K \cdot \pi$ where $K \in \mathbb{Z}$, or $x = (-1)^M \cdot (-\frac{\pi}{6}) + M\pi$ where $M \in \mathbb{Z}$.
Define

$$\begin{aligned} A &= \left\{x \in \mathbb{R} : x = \frac{\pi}{2} + K \cdot \pi \text{ for some } K \in \mathbb{Z}\right\}, \\ B &= \left\{x \in \mathbb{R} : x = (-1)^M \cdot (-\frac{\pi}{6}) + M\pi \text{ for some } M \in \mathbb{Z}\right\}. \end{aligned}$$

The solution set of (†) is given by $A \cup B$.

(b) The general solution of the equation is given by $x = \frac{\pi}{4} + K \cdot \pi$ where $K \in \mathbb{Z}$, or $x = -\frac{\pi}{4} + M\pi$ where $M \in \mathbb{Z}$.
Define

$$\begin{aligned} A &= \left\{x \in \mathbb{R} : x = \frac{\pi}{4} + K \cdot \pi \text{ for some } K \in \mathbb{Z}\right\}, \\ B &= \left\{x \in \mathbb{R} : x = -\frac{\pi}{4} + M\pi \text{ for some } M \in \mathbb{Z}\right\}. \end{aligned}$$

The solution set of the equation is given by $A \cup B$.

(c) The general solution of the equation is given by $x = \frac{K \cdot \pi}{2}$ where $K \in \mathbb{Z}$.
The solution set of the equation is given by $\left\{x \in \mathbb{R} : x = \frac{K \cdot \pi}{2} \text{ for some } K \in \mathbb{Z}\right\}$

(d) The general solution of the equation is given by $x = \frac{K \cdot \pi}{2}$ where $K \in \mathbb{Z}$, or $x = \pm\frac{\pi}{3} + 2M\pi$, where $M \in \mathbb{Z}$.
Define

$$\begin{aligned} A &= \left\{x \in \mathbb{R} : x = \frac{K \cdot \pi}{2} \text{ for some } K \in \mathbb{Z}\right\}, \\ B &= \left\{x \in \mathbb{R} : x = \frac{\pi}{3} + 2M\pi \text{ for some } M \in \mathbb{Z}\right\}, \\ C &= \left\{x \in \mathbb{R} : x = -\frac{\pi}{3} + 2M\pi \text{ for some } M \in \mathbb{Z}\right\} \end{aligned}$$

The solution set of the equation is given by $A \cup B \cup C$.

(e) The general solution of the equation is given by $x = (-1)^K \cdot \frac{\pi}{18} + K \cdot \frac{\pi}{3}$ where $K \in \mathbb{Z}$, or $x = \frac{\pi}{2} + M\pi$ where $M \in \mathbb{Z}$.
Define

$$\begin{aligned} A &= \left\{x \in \mathbb{R} : x = (-1)^K \cdot \frac{\pi}{18} + K \cdot \frac{\pi}{3} \text{ for some } K \in \mathbb{Z}\right\}, \\ B &= \left\{x \in \mathbb{R} : x = \frac{\pi}{2} + M\pi \text{ for some } M \in \mathbb{Z}\right\}. \end{aligned}$$

The solution set of the equation is given by $A \cup B$.

(f) The general solution of the equation is given by $x = \pm\frac{\pi}{9} + K \cdot \frac{2\pi}{3}$ where $K \in \mathbb{Z}$, or $x = \frac{\pi}{2} + M\pi$ where $M \in \mathbb{Z}$.
Define

$$\begin{aligned} A &= \left\{x \in \mathbb{R} : x = \frac{\pi}{9} + K \cdot \frac{2\pi}{3} \text{ for some } K \in \mathbb{Z}\right\}, \\ B &= \left\{x \in \mathbb{R} : x = -\frac{\pi}{9} + L \cdot \frac{2\pi}{3} \text{ for some } L \in \mathbb{Z}\right\}, \\ C &= \left\{x \in \mathbb{R} : x = \frac{\pi}{2} + M\pi \text{ for some } M \in \mathbb{Z}\right\} \end{aligned}$$

The solution set of the equation is given by $A \cup B \cup C$.

(g) Denote by α the number given by $\alpha = \arcsin(\frac{1}{4})$.
The general solution of the equation is given by $x = (-1)^K\alpha + K \cdot \pi$ where $K \in \mathbb{Z}$, or $x = \frac{\pi}{2} + M \cdot 2\pi$ where $M \in \mathbb{Z}$.
Define

$$\begin{aligned} A &= \{x \in \mathbb{R} : x = (-1)^K\alpha + K \cdot \pi \text{ for some } K \in \mathbb{Z}\}, \\ B &= \left\{x \in \mathbb{R} : x = \frac{\pi}{2} + M \cdot 2\pi \text{ for some } M \in \mathbb{Z}\right\} \end{aligned}$$

The solution set of the equation is given by $A \cup B$.

(h) The general solution of the equation is given by $x = (-1)^K \cdot \frac{\pi}{24} + K \cdot \frac{\pi}{4}$ where $K \in \mathbb{Z}$, or $x = \frac{\pi}{2} + M\pi$ where $M \in \mathbb{Z}$.
Define

$$\begin{aligned} A &= \left\{x \in \mathbb{R} : x = (-1)^K \cdot \frac{\pi}{24} + K \cdot \frac{\pi}{4} \text{ for some } K \in \mathbb{Z}\right\}, \\ B &= \left\{x \in \mathbb{R} : x = \frac{\pi}{2} + M\pi \text{ for some } M \in \mathbb{Z}\right\} \end{aligned}$$

The solution set of the equation is given by $A \cup B$.

(i) Denote by α the number given by $\alpha = \arcsin(\frac{5}{13})$.

The general solution of the equation is given by $x = -\frac{\alpha}{3} + N \cdot \frac{2\pi}{3}$ where $N \in \mathbb{Z}$.

The solution set of the equation is given by $\left\{x \in \mathbb{R} : x = -\frac{\alpha}{3} + N \cdot \frac{2\pi}{3} \text{ for some } N \in \mathbb{Z}\right\}$.

(j) Denote by α the number given by $\alpha = \arcsin(\frac{1}{4})$.

The general solution of the equation is given by $x = -(-1)^N \cdot \frac{\alpha}{6} + N \cdot \frac{\pi}{6}$ where $N \in \mathbb{Z}$.

The solution set of the equation is given by $\left\{x \in \mathbb{R} : x = -(-1)^N \cdot \frac{\alpha}{6} + N \cdot \frac{\pi}{6} \text{ for some } N \in \mathbb{Z}\right\}$.

(k) The general solution of the equation is given by $x = \frac{\pi}{9} + M \cdot \frac{2\pi}{3}$, where M is an arbitrary integer, or $x = \frac{\pi}{2} + N \cdot \pi$, where N is an arbitrary integer.
Define

$$\begin{aligned} A &= \left\{x \in \mathbb{R} : x = \frac{\pi}{9} + M \cdot \frac{2\pi}{3} \text{ for some } M \in \mathbb{Z}\right\}, \\ B &= \left\{x \in \mathbb{R} : x = \frac{\pi}{2} + N \cdot \pi \text{ for some } N \in \mathbb{Z}\right\} \end{aligned}$$

The solution set of the equation is given by $A \cup B$.

(l) The general solution of the equation is given by $x = \frac{12}{6N + (-1)^N} \cdot \frac{1}{\pi}$, where N is an integer.

The solution set of the equation is given by $\left\{x \in \mathbb{R} : x = \frac{12}{6N + (-1)^N} \cdot \frac{1}{\pi} \text{ for some } N \in \mathbb{Z}\right\}$.

(m) The general solution of the equation is given by $x = \sqrt[3]{\frac{\pi}{2} + N \cdot 3\pi}$, where N is an integer.

The solution set of the equation is given by $\left\{x \in \mathbb{R} : x = \sqrt[3]{\frac{\pi}{2} + N \cdot 3\pi} \text{ for some } N \in \mathbb{Z}\right\}$.

(n) The general solution of the equation is given by $x = \pm\sqrt{N} \cdot \frac{\sqrt{\pi}}{2}$ where $N \in \mathbb{N}$.

Define

$$\begin{aligned} A &= \left\{x \in \mathbb{R} : x = \sqrt{N} \cdot \frac{\sqrt{\pi}}{2} \text{ for some } N \in \mathbb{N}\right\}, \\ B &= \left\{x \in \mathbb{R} : x = -\sqrt{N} \cdot \frac{\sqrt{\pi}}{2} \text{ for some } N \in \mathbb{N}\right\}. \end{aligned}$$

The solution set of the equation is given by $A \cup B$.

(o) The general solution of the equation is given by $x = \frac{1}{K^2 \cdot \pi^2}$ where $K \in \mathbb{N}\backslash\{0\}$, or $x = \pm\sqrt{M} \cdot \sqrt{\pi}$ where $M \in \mathbb{N}\backslash\{0\}$.
Define

$$\begin{aligned} A &= \left\{x \in \mathbb{R} : x = \frac{1}{K^2 \cdot \pi^2} \text{ for some } K \in \mathbb{N}\backslash\{0\}\right\}, \\ B &= \left\{x \in \mathbb{R} : x = \sqrt{M} \cdot \sqrt{\pi} \text{ for some } M \in \mathbb{N}\backslash\{0\}\right\} \\ C &= \left\{x \in \mathbb{R} : x = -\sqrt{M} \cdot \sqrt{\pi} \text{ for some } M \in \mathbb{N}\backslash\{0\}\right\}. \end{aligned}$$

The solution set of the equation is given by $A \cup B \cup C$.

(p) The general solution of the equation is given by $x = \frac{1}{K \cdot \pi}$ where $K \in \mathbb{Z} \backslash \{0\}$, or $x = \frac{3}{6M \pm 1} \cdot \frac{1}{\pi}$ where $M \in \mathbb{Z}$. Define

$$\begin{aligned} A &= \left\{ x \in \mathbb{R} : x = \frac{1}{K \cdot \pi} \text{ for some } K \in \mathbb{Z} \backslash \{0\} \right\}, \\ B &= \left\{ x \in \mathbb{R} : x = \frac{3}{6M+1} \cdot \frac{1}{\pi} \text{ for some } M \in \mathbb{Z} \right\}, \\ C &= \left\{ x \in \mathbb{R} : x = \frac{3}{6M-1} \cdot \frac{1}{\pi} \text{ for some } M \in \mathbb{Z} \right\}. \end{aligned}$$

The solution set of the equation is given by $A \cup B \cup C$.

19. The general solution of the equation is given by $x = \frac{\pi}{10} + K \cdot \frac{2\pi}{5}$ where $K \in \mathbb{Z}$, or $x = M\pi$ where $M \in \mathbb{Z}$.

20. The general solution of the equation is given by $x = \frac{3\pi}{20} + K \cdot \frac{\pi}{5}$ where $K \in \mathbb{Z}$, or $x = \frac{\pi}{2} + M\pi$ where $M \in \mathbb{Z}$.

21. The general solution of the equation is given by $x = 2K\pi$ where $K \in \mathbb{Z}$, or $x = \frac{\pi}{5} + L \cdot \frac{2\pi}{5}$ where $L \in \mathbb{Z}$, or $x = \frac{\pi}{2} + M\pi$ where $M \in \mathbb{Z}$.

22. The general solution of the equationis given by $x = \pm\frac{\pi}{9} + K \cdot \frac{2\pi}{3}$ where $K \in \mathbb{Z}$, or $x = M\pi$ where $M \in \mathbb{Z}$.

23. (a) i. ——
    ii. ——
    (b) i. ——
    ii. $k \leq \frac{1}{3}$ or $k \geq 3$.
    (c) The general solution of the equation is given by $x = \frac{\pi}{36} + (-1)^N \mu + N\pi$, where $N \in \mathbb{Z}$.

24. $z = \cos(\frac{\pi}{6}) + i\sin(\frac{\pi}{6})$.
    $\arg(z^{2018}) = \frac{\pi}{3}$.

25. (a) $\sqrt[4]{2}(\cos(\frac{\pi}{8}) + i\sin(\frac{\pi}{8}))$, $\sqrt[4]{2}(\cos(-\frac{7\pi}{8}) + i\sin(-\frac{7\pi}{8}))$.
    (b) $\sqrt[6]{2}(\cos(\frac{\pi}{12}) + i\sin(\frac{\pi}{12}))$, $\sqrt[6]{2}(\cos(\frac{3\pi}{4}) + i\sin(\frac{3\pi}{4}))$, $\sqrt[6]{2}(\cos(-\frac{7\pi}{12}) + i\sin(-\frac{7\pi}{12}))$.
    (c) $\sqrt[8]{2}(\cos(\frac{\pi}{16}) + i\sin(\frac{\pi}{16}))$, $\sqrt[8]{2}(\cos(\frac{9\pi}{16}) + i\sin(\frac{9\pi}{16}))$, $\sqrt[8]{2}(\cos(-\frac{15\pi}{16}) + i\sin(-\frac{16\pi}{16}))$, $\sqrt[8]{2}(\cos(-\frac{7\pi}{16}) + i\sin(-\frac{7\pi}{16}))$.

26. $(\cos(-\frac{\pi}{18}) + i\sin(-\frac{\pi}{18}))$, $(\cos(\frac{11\pi}{18}) + i\sin(\frac{11\pi}{18}))$, $(\cos(-\frac{13\pi}{18}) + i\sin(-\frac{13\pi}{18}))$.

27. (a) The general solution of the equation $\cos(x) = \frac{1}{\sqrt{2}}$ is $x = \pm\frac{\pi}{4} + 2N\pi$, where N is an integer.
    (b) i. ——
    ii. $\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^m + \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)^m = \zeta^m + \bar{\zeta}^m = 2r^m\cos(m\theta) = 2\cos(m\theta)$.
    iii. $\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^m + \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)^m = \sqrt{2}$ iff ($m = \pm 1 + 8N$ for some integer N).

28. (a) $(1+i)^p - (1-i)^p = 0$ iff ($p = 4N$ for some $N \in \mathbb{Z}$).
    (b) 2.

29. (a) $\omega = \cos(\frac{2\pi}{3}) + i\sin(\frac{2\pi}{3})$ or $\omega = \cos(-\frac{2\pi}{3}) + i\sin(-\frac{2\pi}{3})$.
    (b) i. ——
    ii. $\alpha^3 = 1 = \beta^3$.
    iii. $1, -1, 2$.

30. (a) i. ——
    ii. $\frac{(z^2 - 1/z^2)i}{z^2 + 1/z^2} = -\sqrt{3}$ iff ($z = i^M\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)$ for some $M \in \mathbb{Z}$).
    (b) i. $\cos(\frac{\pi}{3}) + i\sin(\frac{\pi}{3})$, $\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3})$.
    ii.

A. x^2-2x+1. B. x^2+x+1. C. x^2+x+1.

31. (a) ——

(b) i. $\cos^7(\theta)=\dfrac{1}{2^6}(\cos(7\theta)+7\cos(5\theta)+21\cos(3\theta)+35\cos(\theta))$.

$\sin^7(\theta)=\dfrac{1}{2^6}(-\sin(7\theta)+7\sin(5\theta)-21\sin(3\theta)+35\sin(\theta))$.

ii. $\cos^8(\theta)=\dfrac{1}{2^7}(\cos(8\theta)+8\cos(6\theta)+28\cos(4\theta)+56\cos(2\theta)+35)$.

$\sin^8(\theta)=\dfrac{1}{2^7}(\cos(8\theta)-8\cos(6\theta)+28\cos(4\theta)-56\cos(2\theta)+35)$.

32. ——

33. (a) ——

(b) ——

(c) i. $\omega=\dfrac{-1+\sqrt{5}}{4}+\dfrac{\sqrt{10+2\sqrt{5}}}{4}i$.

ii. $\omega^2=\dfrac{-1-\sqrt{5}}{4}+\dfrac{\sqrt{10-2\sqrt{5}}}{4}i$.

iii. $\omega^3=\overline{\omega^2}=\dfrac{-1-\sqrt{5}}{4}-\dfrac{\sqrt{10-2\sqrt{5}}}{4}i$, and $\omega^4=\bar{\omega}=\dfrac{-1+\sqrt{5}}{4}-\dfrac{\sqrt{10+2\sqrt{5}}}{4}i$.

34. ——

35. ——

36. ——

37. ——

38. ——

39. ——

40. ——

41. ——

42. ——

43. (a) i. $A\subset\mathbb{Q}$. ii. $\mathbb{Q}\not\subset A$.

(b) i. $B\subset A$. ii. $A\not\subset B$.

(c) i. $C\subset A$. ii. $A\not\subset C$.

(d) i. $D\subset A$. ii. $A\not\subset D$.

44. (a) It is not true that $B\subset A$. (b) It is not true that $A\subset B$.

45. ——

46. ——

47. ——

48. ——