MATH1050 Exercise 5 Supplement (Answers)

1. (a) $A = B = 1$.
   (b) ——
2. (a) $A = B = 1$.
   (b) ——
3. (a) $k_1 = 6n$, $k_2 = 6an + 3n(6n-1)$, $k_3 = 6an(6n-1) + n(6n-1)(6n-2)$.
   (b) i. $A = -18$, $B = 27$, $C = -7$.
      ii. $n = 1$.
4. *Hint.* In terms of m, n, the coefficient a of the x-term and the coefficient b of the x^2-term are respectively given by $a = m$ and $b = \dfrac{m(m-1)}{2} - (mn - m) = \dfrac{m(m-2n+1)}{2}$.
5. ——
6. ——
7. $A = 9$, $B = 1$.
8. ——
9. (a) $\displaystyle\sum_{j=0}^{n} \binom{n}{j}^2 = \binom{2n}{n}$.

   Remark. Make use of the equality $(1+x)^{2n} = (1+x)^n(1+x)^n$ as polynomials. Express the coefficient of x^n in two different ways, one according to one side of this equality.

   (b) $\displaystyle\sum_{j=0}^{n} (-1)^j \binom{n}{j}^2 = \begin{cases} (-1)^{n/2}\binom{n}{n/2} & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$.

   Remark. Make use of the equality $(1-x^2)^n = (1+x)^n(1-x)^n$ as polynomials. Express the coefficient of x^n in two different ways, one according to one side of this equality.
10. (a) $A = 2$, $B = 4$, $C = 3$.
    (b) $A = 3$, $B = 3$, $C = 8$.
11. (a) i. ——
       ii. A. $\binom{n+m+1}{k+1} - \binom{n}{k+1}$.
           B. $\binom{n+m+1}{k+1}$.
    (b) i. ——
       ii. $24\binom{m+5}{5}$
12. ——
13. ——
14. (a) i. ——
       ii. $f'(0) = \begin{cases} 0 & n \text{ is odd} \\ (-1)^{n/2}(n!) & n \text{ is even} \end{cases}$
    (b) ——
15. ——
16. ——
17. ——
18. ——