MATH1050 Exercise 3 Supplement (Answers)

1. *Hint.* Always refer to the definitions for the respective notions of real part, imaginary part, complex conjugate and modulus.

2. (a) $\omega^2 = i$, $\omega^8 = 1$, $\omega^{2016} = \omega^{252 \cdot 8} = 1$.

   (b) $2 + \sqrt{2}$.

3. $\zeta + \bar{\zeta} = 2\mathsf{Re}(\zeta) = 2a$.

   $\zeta^2 + \bar{\zeta}^2 = 4a^2 - 2r^2$.

   $\zeta^3 + \bar{\zeta}^3 = 8a^3 - 6ar^2$.

   $\zeta^4 + \bar{\zeta}^4 = 16a^4 - 16a^2r^2 + 2r^4$.

   $\zeta^5 + \bar{\zeta}^5 = 32a^5 - 40a^3r^2 + 10ar^4$.

   $\zeta^6 + \bar{\zeta}^6 = 64a^6 - 96a^4r^2 + 36a^2r^4 - 2r^6$.

   Remark. One possible approach is to make good use of binomial expansions.

4. (a) $\mathsf{Re}(\zeta) = 2k^2 - 3k - 2$ and $\mathsf{Im}(\zeta) = k^2 - 3k + 2$.

   i. One possibility is $k = 1$ and $\zeta = -3$.
   The other is $k = 2$ and $\zeta = 0$.

   ii. One possibility is $k = -\dfrac{1}{2}$ and $\zeta = \dfrac{15}{4}i$. The other is $k = 2$ and $\zeta = 0$.

   iii. $k = -2$ and $\zeta = 12 + 12i$.

5. ——

6. (a) ——

   (b) $z = \left(\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i\right) w$ or $z = \left(\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i\right) w$.

7. (a) 1

   (b) ——

8. ——

9. ——

10. $a = -\dfrac{7}{2}$ and $b = \dfrac{1}{2}$.

11. $a = -2$ and $b = 2$.

12. *Hint.* Make use of the relations between the coefficients of $f(x)$ (possibly together with the discriminant of $f(x)$) and the sum of roots, the product rules.

13. (a) ——

    (b) $z = 0$ or $z = 1$ or $z = -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i$ or $z = -\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i$.

14. $z = 1 + \sqrt{3}i$ or $z = 1 - \sqrt{3}i$.

15. The point on the circle C in the Argand plane which is of the minimum distance from p is $(2 + 2.5\sqrt{2}) + (-3 - 2.5\sqrt{2})i$.

    The point on the circle C in the Argand plane which is of the maximum distance from p is $(2 - 2.5\sqrt{2}) + (-3 + 2.5\sqrt{2})i$.

16. $2 + 3i$.

17. (a) $\mathsf{Im}(z) = -\frac{1}{2}\mathsf{Re}(z) + 5$.

    (b) $2\sqrt{5}$.

18. $-1 - i = \sqrt{2}\Big(\cos(-\dfrac{3\pi}{4}) + i\sin(-\dfrac{3\pi}{4})\Big)$.

    $1 - i = \sqrt{2}\Big(\cos(-\dfrac{\pi}{4}) + i\sin(-\dfrac{\pi}{4})\Big)$.

    $\dfrac{-1 - i}{(1 - i)^5} = \dfrac{i}{4}$

19. -3.

20. (a) $p = \sqrt{3} + i$.

    $r = -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i$.

    (b) $q = \dfrac{2\sqrt{3} - 1}{2} + \dfrac{2 + \sqrt{3}}{2}i$.

21. (a) *Geometric interpretation of the result on the Argand plane* The points α, σ both lies on the unit circle with centre 0. The distance between 1 and σ and the same as the distance between α and σ. $0, 1, \sigma$ are the three vertices of a isosceles triangle with base being the line segment joining 1 and σ. $0, \alpha, \sigma$ are the three vertices of an isosceles triangle with base being the line segment joining α and σ. These two isosceles triangle are congruent to each other. Hence the angle subtended by the line segment joining 0 and 1 and the line segment joining 0 and σ is the same as the angle subtended by the line segment joining 0 and σ and the line segment joining 0 and α. The line which joins 0 and σ bisects the angle subtended by the line segment joining 0 and 1 and the line segment joining 0 and α.

    (b) i. $\dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}i$, $-\dfrac{1}{\sqrt{2}} - \dfrac{1}{\sqrt{2}}i$.

    ii. $\dfrac{1}{\sqrt{2}} - \dfrac{1}{\sqrt{2}}i$, $-\dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}i$.

    iii. $\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i$, $-\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i$.

    iv. $\dfrac{\sqrt{3}}{2} + \dfrac{\sqrt{1}}{2}i$, $-\dfrac{\sqrt{3}}{2} - \dfrac{\sqrt{1}}{2}i$.

    v. $\dfrac{\sqrt{\sqrt{10} + 1}}{\sqrt{2}} + \dfrac{\sqrt{\sqrt{10} - 1}}{\sqrt{2}}i$,
    $-\dfrac{\sqrt{\sqrt{10} + 1}}{\sqrt{2}} - \dfrac{\sqrt{\sqrt{10} - 1}}{\sqrt{2}}i$.

    vi. $\dfrac{\sqrt{\sqrt{2} + 1}}{\sqrt{2}} + \dfrac{\sqrt{\sqrt{2} - 1}}{\sqrt{2}}i$,
    $-\dfrac{\sqrt{\sqrt{2} + 1}}{\sqrt{2}} - \dfrac{\sqrt{\sqrt{2} - 1}}{\sqrt{2}}i$.

22. (a) $z = 0$ or $z = -4i$.

(b) $z=-3$ or $z=2i$.

(c) $z=(1+\sqrt{2})i$ or $z=(1-\sqrt{2})i$.

(d) $z=2+\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{2}}i$ or $z=2+\dfrac{1}{\sqrt{2}}-\dfrac{1}{\sqrt{2}}i$.

(e) $z=3+i$ or $z=-1+i$.

(f) $z=2i$ or $z=3i$.

23. (a) i. $\dfrac{3\sqrt{2}}{2}+\dfrac{\sqrt{2}}{2}i, -\dfrac{3\sqrt{2}}{2}-\dfrac{\sqrt{2}}{2}i$.

ii. $\dfrac{3\sqrt{2}}{2}-\dfrac{\sqrt{2}}{2}i, -\dfrac{3\sqrt{2}}{2}+\dfrac{\sqrt{2}}{2}i$.

(b) $x=\dfrac{3\sqrt{2}}{2}+\dfrac{\sqrt{2}}{2}i$ or $x=-\dfrac{3\sqrt{2}}{2}-\dfrac{\sqrt{2}}{2}i$ or $x=\dfrac{3\sqrt{2}}{2}-\dfrac{\sqrt{2}}{2}i$ or $x=-\dfrac{3\sqrt{2}}{2}+\dfrac{\sqrt{2}}{2}i$.

24. (a) 2.

(b) $\sqrt{2}+\sqrt{2}i$ or $\sqrt{2}-\sqrt{2}i$ or $-\sqrt{2}+\sqrt{2}i$ or $-\sqrt{2}-\sqrt{2}i$.

25. ——

26. (a) $A=2$.

(b) i. 0.

ii. $B=C=D=1$.

27. ——

28. ——

29. ——