1. **Real-valued functions of one real variable in school mathematics**.
   Below is a typical 'explanation' of the notion of real valued functions of one real variable in school mathematics:

   Let D be a subset of $\mathbb{R}$ (very often $\mathbb{R}$ itself or $\mathbb{R}$ with a few points deleted). A real-valued function defined on D is a 'rule of assignment' from D to $\mathbb{R}$, so that each number in D is being assigned to exactly one element of $\mathbb{R}$.
   When we refer to such a function by f, the set D will be referred to as the domain of this function f.
   Whenever $x \in D$, $y \in \mathbb{R}$ and x is assigned to y, we write $y = f(x)$.
   The set $G = \{(x, f(x)) \mid x \in D\}$ will be called the graph of f. Note that $G \subset \mathbb{R}^2$.

2. **In-formal definition of function**.
   Let A, B be sets. A function from A to B is a 'rule of assignment' from A to B, so that each element of A is being assigned to exactly one element of B.

   Conventions and notations.

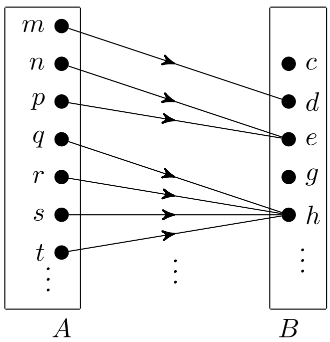
   - When we denote such a function by f, we refer to it as $f : A \longrightarrow B$. Whenever $x \in A$, $y \in B$ and x is assigned to y, we write $y = f(x)$ (or $x \underset{f}{\mapsto} y$).
   - A is called the **domain** of f. B is called the **range** of f.

   Remark. We postpone the generalization of the notion of graphs of functions.

3. **'Blobs-and-arrows diagrams'.**
   We may visualize a function by its '**blobs-and-arrows diagram**'.
   We illustrate the idea with the example below:
   Let $A = \{m, n, p, q, r, s, t, ...\}$, $B = \{c, d, e, g, h, ...\}$, and $f : A \longrightarrow B$ be defined by

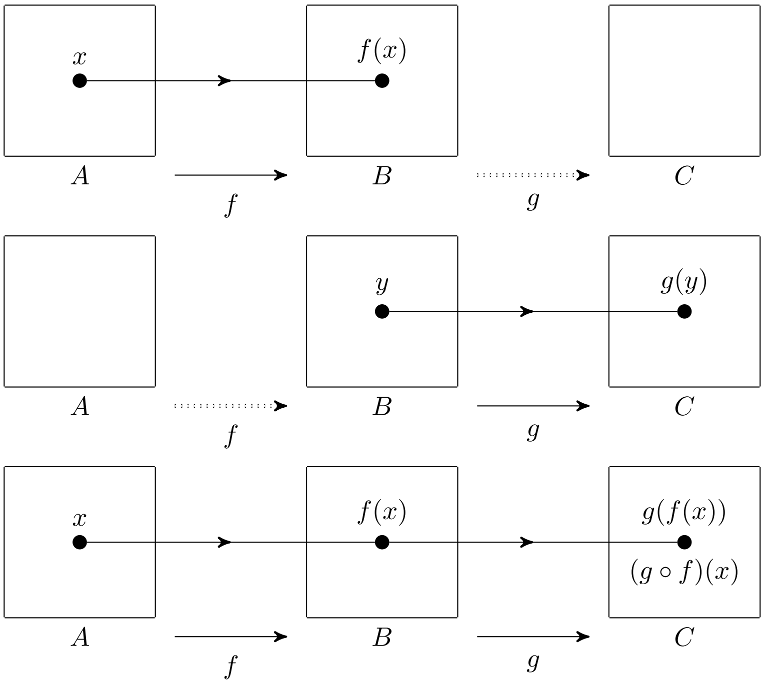
$$f(m) = d, f(n) = e, f(p) = e, f(q) = h, f(r) = h, f(s) = h, f(t) = h, \cdots .$$

   By definition, f assigns m to d, n to e, p to e, q to h, r to h, s to h, t to h, $\cdots$.
   We draw the 'blobs-and-arrows diagram' of the function f as:



4. **Notion of equality for functions.**
   We regard two functions to be the same as each other exactly when they 'determine the same assignment'.
   Definition.
   Let A_1, A_2, B_1, B_2 be sets, and $f_1 : A_1 \longrightarrow B_1$, $f_2 : A_2 \longrightarrow B_2$ be functions.
   *We agree to say that f_1 is **equal** to f_2 as functions, and to write $f_1 = f_2$, exactly when $A_1 = A_2$ and $B_1 = B_2$ and $f_1(x) = f_2(x)$ for any $x \in A_1$.*

5. **Compositions.**
   Out of two functions, the range of one being the same as the domain of the other, we may construct a third function.
   Definition.
   Let A, B, C be sets, and $f : A \longrightarrow B$, $g : B \longrightarrow C$ be functions.
   Define the function $g \circ f : A \longrightarrow C$ by $(g \circ f)(x) = g(f(x))$ for any $x \in A$.
   *$g \circ f$ is called the **composition** of the functions f, g.*



Lemma (1). (Associativity of composition.)

Let A, B, C, D be sets, and $f : A \longrightarrow B$, $g : B \longrightarrow C$, $h : C \longrightarrow D$ be functions. $(h \circ g) \circ f = h \circ (g \circ f)$ as functions.

Remark. Hence there is no ambiguity when we refer to $(h \circ g) \circ f$ (and $h \circ (g \circ f)$) as $h \circ g \circ f$.

Proof of Lemma (1).

Let A, B, C, D be sets, and $f : A \longrightarrow B$, $g : B \longrightarrow C$, $h : C \longrightarrow D$ be functions.

Note that $(h \circ g) \circ f$, $h \circ (g \circ f)$ have the same domain, namely, A.

Also note that $(h \circ g) \circ f$, $h \circ (g \circ f)$ have the same range, namely, D.

[We want to verify: For any $x \in A$, $((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$.]

Pick any $x \in A$. $((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x))) = h((g \circ f)(x)) = (h \circ (g \circ f))(x)$.

It follows that $(h \circ g) \circ f = h \circ (g \circ f)$ as functions.

Warning. Suppose A, B are sets and $f : A \longrightarrow B$, $g : B \longrightarrow A$ are functions. Then it makes sense to construct the functions $g \circ f$, $f \circ g$. However, $g \circ f$, $f \circ g$ are not necessarily equal to each other. Even when $A = B$, these two functions $g \circ f$, $f \circ g$ are still not necessarily equal to each other.

6. **Identity function, inclusion function, restrictions and extensions.**

Here are the formal definitions (in terms of set language) of several miscellaneous notions used in various occasions.

Definition.
Let A be a set.

(a) *Define the function $\mathsf{id}_A : A \longrightarrow A$ by $\mathsf{id}_A(x) = x$ for any $x \in A$. id_A is called* **the identity function on** A.

(b) *Let S be a subset of A. Define the function $\iota_S^A : S \longrightarrow A$ by $\iota_S^A(x) = x$ for any $x \in S$. ι_S^A is called the* **inclusion function of S into A.**

Definition.
Let A, B be sets, and $f : A \longrightarrow B$ be a function.

(a) *Let S be a subset of A.*

The function $f \circ \iota_S^A : S \longrightarrow B$ is called the **restriction of f to S**. *It is denoted by $f|_S$.*

(b) *Let H be a set which contains A as a subset, K be a set which contains B as a subset.*

Suppose $g : H \longrightarrow K$ be a function which satisfies $g \circ \iota_A^H = \iota_B^K \circ f$. Then g is called an **extension of f**.

7. **Graphs of functions.**

To generalize the notion of graphs of functions, we need bring in the notion of cartesian products for two arbitrary sets.

Definition.

Let A, B be sets, and $f : A \longrightarrow B$ be a function. Define $G = \{(x, f(x)) \mid x \in A\}$.

G is called the **graph of the function** *f. Note that $G \subset A \times B$.*

Lemma (2). (Equality of functions and equality of graphs.)

Let A, B be sets, and $f_1, f_2 : A \longrightarrow B$ be functions. Suppose G_1, G_2 are the respective graphs of f_1, f_2. Then f_1 is equal to f_2 as functions iff $G_1 = G_2$.

Proof of Lemma (2). Exercise in set language.

8. **'Coordinate plane diagrams'.**

We may visualize a function, displaying its graph, by its '**coordinate plane diagram**'.

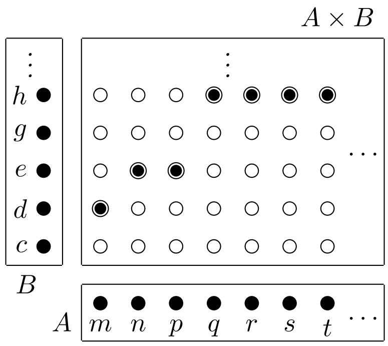
We illustrate the idea with the example below:

Let $A = \{m, n, p, q, r, s, t, ...\}$, $B = \{c, d, e, g, h, ...\}$, and $f : A \longrightarrow B$ be defined by

$$f(m) = d, f(n) = e, f(p) = e, f(q) = h, f(r) = h, f(s) = h, f(t) = h, \cdots .$$

By definition, the graph of f is the set $G = \{(m, d), (n, e), (p, e), (q, h), (r, h), (s, h), (t, h), \cdots\}$.

We draw the 'coordinate plane diagram' of the function f as:



9. **'Blobs-and-arrows diagram' versus 'coordinate plane diagram'.**

Depending on how we like the 'information' concerned with a given function $f : A \longrightarrow B$ is presented, we may draw its '**coordinate plane diagram**' or its '**blobs-and-arrows diagram**'. Each has its own advantage.

- In the 'coordinate plane diagram', the graph of f is displayed.
- In the 'blobs-and-arrows diagram', the visual picture of f as a 'rule of assignment' is emphasized.

The two diagrams may be converted from one to the other in a systematic way. We illustrate the idea with the example below:

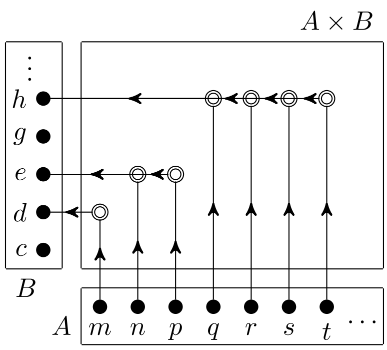
Let $A = \{m, n, p, q, r, s, t, ...\}$, $B = \{c, d, e, g, h, ...\}$, and $f : A \longrightarrow B$ be defined by

$$f(m) = d, f(n) = e, f(p) = e, f(q) = h, f(r) = h, f(s) = h, f(t) = h, \cdots .$$

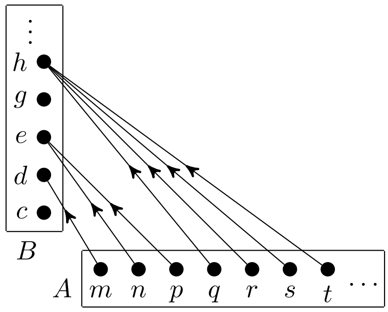
By definition, f assigns m to d, n to e, p to e, q to h, r to h, s to h, t to h, $\cdots$.

The graph of f is the set $G = \{(m, d), (n, e), (p, e), (q, h), (r, h), (s, h), (t, h), \cdots\}$.

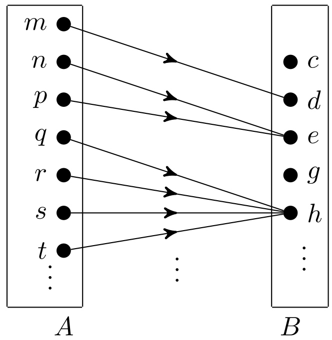
(a) *'Coordinate plane diagram' of f:*



(b) *In-between the two kinds of diagrams:*



(c) *'Blobs-and-arrows diagram' of f:*



10. **Basic examples of functions in school maths and beyond.**

We have encountered various examples of functions in school mathematics and in basic MATH courses.

(a) **Polynomial functions with real coefficients.**

Such functions are: the zero function; non-zero constant functions; linear functions; quadratic (polynomial) functions; cubic functions; et cetera.

- Suppose $n \in \mathbb{N}$, and $a_0, a_1, a_2, \cdots, a_n \in \mathbb{R}$, with $a_n \neq 0$.
  The polynomial function naturally defined by the degree-n polynomial $a_n x^n + \cdots + a_2 x^2 + a_1 x + a_0$ with indeterminate x is the function with domain and range both being $\mathbb{R}$ and which assigns each real number c to the real number $a_n c^n + \cdots + a_2 c^2 + a_1 c + a_0$.
  When we refer to the polynomial as $p(x)$, we will usually also choose to refer to the polynomial function thus defined as p. When we refer to the polynomial function as p, we will also refer to the polynomial itself as $p(x)$.
  In school maths we very often deliberately confuse the polynomial expression $p(x)$ with the function p.

But we have to be careful from now on. When we talk about a function, we have to clarify its domain and its range.

(b) **Rational functions with real coefficients.**

Such a function is one whose 'formula of definition' is given by a fraction whose numerator and denominator are both polynomial with one indeterminate and with real coefficients. Its range is $\mathbb{R}$, and its domain is $\mathbb{R}\backslash Z$, in which Z is the set of real roots of the polynomial in the denominator of the fraction.

Polynomial functions are regarded as rational functions.

In school maths we very often deliberately confuse the rational function with the fraction of the polynomials which defines the function. But from now on, we have to be more careful.

(c) **'Algebraic functions' in school maths.**

An explicit 'algebraic function' is one whose 'formula of definition' is obtained from polynomials upon 'finitely many' operations with $+, -, \times, \div$ and taking surds. Its domain is likely to be an interval, or a 'disjoint union' of several intervals. Its range is $\mathbb{R}$.

Rational functions are regarded as explicit 'algebraic functions'.

But there are implicit 'algebraic function' as well: they are the functions 'implicitly defined' by polynomial equations with two unknowns, say, x, y with real coefficients. You have already encountered such objects when you learnt 'implicit differentiation' in *calculus of one real variable.*

(d) **Elementary transcendental functions.**

These are $\exp, \ln, \cos, \sin, \tan, \sec, \csc, \cot$, and other functions which can be obtained from them upon 'finitely many' operations with $+, -, \times, \div$ and taking surds.

The domain of such a function is usually an interval or a 'disjoint union' of several intervals. Its range is $\mathbb{R}$.

You learnt calculus at school because these functions are not easy to study with purely algebraic means.

(e) **'Multivariable functions' in multivariable calculus.**

In your *advanced calculus* courses, you encounter '$\mathbb{R}^m$-valued functions of n real variables': functions from $D \subset \mathbb{R}^n$ to $\mathbb{R}^m$. Here D is usually a 'nice' subset of $\mathbb{R}^n$.

(f) **Functions of one complex variable.**

In your *complex variables* course, you will encounter 'complex-valued functions of one complex variable'. Such a function has range $\mathbb{C}$. Its domain is usually a 'nice' subset of $\mathbb{C}$, in the sense that it is open in $\mathbb{C}$ and connected.

Basic examples of such 'complex-valued functions of one complex variable' are:

- polynomial functions with complex coefficients, whose 'formulae of definition' are polynomials with complex coefficients,
- rational functions with complex coefficients, whose 'formulae of definition' are fractions of polynomials with complex coefficients, and
- elementary transcendental functions $\exp, \ln, \cos, \sin, \tan, \sec, \csc, \cot$ et cetera.

(g) **Infinite sequences and families.**

An infinite sequence of real numbers, say, $\{a_n\}_{n=0}^{\infty}$ can be thought as a function whose domain and range are $\mathbb{N}$ and $\mathbb{R}$ respectively, and which assigns each natural number n to the real number a_n.

The idea can be extended to the notion of infinite sequences in an arbitrary set and further to the notion of 'family'.

(h) **'Algebraic operations' for algebraic structures.**

In your *algebra* course, you will encounter various 'algebraic operations'. They are functions with special properties which give rise to various 'algebraic structures'.

The prototype of these 'algebraic operations' are the 'arithmetic operations' addition, subtraction, multiplication, division in school maths. Below are some examples:

i. 'Addition' for natural numbers can be regarded as the function whose domain is $\mathbb{N}^2$ and whose range is $\mathbb{N}$, assigning each ordered pair of natural numbers (x, y) to the sum $x + y$.

ii. 'Multiplication' for real numbers can be regarded as the function whose domain is $\mathbb{R}^2$ and whose range is $\mathbb{R}$, assigning each ordered pair of real numbers (x, y) to the sum $x \times y$.

iii. 'Conjugation' for complex numbers can be regarded as the function whose domain is $\mathbb{C}$ and whose range is $\mathbb{C}$, assigning each complex number ζ to its complex conjugate $\bar{\zeta}$.

For more detail, refer to the handout *Abelian groups, integral domains and fields*.

(i) **Linear transformations and 'transformation for various algebraic structures'.**

The mathematical objects under consideration in your *linear algebra* course are vector spaces and linear transformations.

A linear transformation is a function whose domain and range are vector spaces and which 'preserves' 'linear structure'.

The prototype of linear transformations is the operation 'matrix multiplication to column vectors from the left':

- Suppose A is an $(m \times n)$-matrix with real entries. Define the function $L_A : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ by $L_A(\mathbf{x}) = A\mathbf{x}$ for any $\mathbf{x} \in \mathbb{R}^n$. The function L_A is called the linear transformation defined by matrix multiplication from the left by A.

  It 'preserves' 'linear structure' in the sense that if a certain linear relation for a collection of vectors in the domain $\mathbb{R}^n$ holds, then the corresponding linear relation in $\mathbb{R}^m$ resultant from replacing the vectors in $\mathbb{R}^n$ by their respective images in $\mathbb{R}^m$ under L_A will hold as well.

For more detail, refer to the handout *Linear algebra beyond matrices and vectors*.

Similar ideas (about 'structure-preserving functions') will be employed in the study of various algebraic structures in your *algebra* courses.

(j) **Various 'operations' in calculus and beyond.**

Differentiation and integration can be thought of as functions.

Here are three simple examples:

For each interval J, denote by $C^1(J)$ the set of all continuously differentiable functions on J and $C(J)$ the set of all continuous function on J.

i. *Let $a, b \in \mathbb{R}$. Suppose $a < b$. Differentiation assigns each continuously differentiable function φ on the open interval (a, b) to its derivative φ'. Note that φ' is a continuous function on (a, b).*

   Hence this 'operation' defines the function D from $C^1((a, b))$ to $C((a, b))$ by $D(\varphi) = \varphi'$ for any $\varphi \in C^1((a, b))$.

ii. *Let $a, b \in \mathbb{R}$. Definite integration with lower limit a and upper limit b assigns each continuous function ψ on the interval $[a, b]$ to the number* $\displaystyle\int_a^b \psi$.

Hence this 'operation' defines the function I_a^b from $C([a, b])$ to $\mathbb{R}$ by $I_a^b(\psi) = \displaystyle\int_a^b \psi$ *for any* $\psi \in C([a, b])$.

iii. *Let $a \in \mathbb{R}$. According to the Fundamental Theorem of the Calculus, definite integration with lower limit a assigns each continuous function ψ on $\mathbb{R}$ to its primitive on $\mathbb{R}$ which vanishes at a.*

Hence this 'operation' defines the function I_a from $C(\mathbb{R})$ to $C^1(\mathbb{R})$ by $(I_a(\psi))(x) = \displaystyle\int_a^x \psi$ *for any* $\psi \in C(\mathbb{R})$ *for any* $x \in \mathbb{R}$.

This viewpoint in seeing differentiation, integration et cetera is useful in some advanced courses.
(Laplace transforms and Fourier transforms can also be seen in this light.)

11. **Families.**
The notion of infinite sequences of real numbers can be generalized to the notion of families for collections of objects.

Definition.
Suppose I, B are sets, and $\varphi : I \longrightarrow B$ is a function.

Then we say φ is a **family in B, indexed by I.**

The set I is referred to as the **index set** for this family, and the set $\{x \in B : x = \varphi(t) \text{ for some } t \in I\}$ is called the **set of all terms** for this family.

Remarks.

(a) This definition generalizes the notion of infinite sequences of real numbers. When $I = \mathbb{N}$, a family in B is just an infinite sequence in B; when furthermore $B = \mathbb{R}$, it is just an infinite sequence of real numbers.

(b) We imitate the notations for infinite sequences when we regard the function φ from I to B as a family in B with index set I. We suppress the symbols φ, B, and present this family as, say, $\{x_\alpha\}_{\alpha \in I}$, in which the symbol x_α stands for $\varphi(\alpha)$ for each $\alpha \in I$.

(c) What is the point of this definition?
It gives us the flexibility to regard the same family in B, say, $\{x_\alpha\}_{\alpha \in I}$, as a family in C which contains B as a subset, whenever it is convenient for us to do so.

Examples.

(a) $\{(-\infty, u)\}_{u \in \mathbb{R}}$ stands for the function with domain $\mathbb{R}$ and range, say, the set of all intervals, assigning each real number u to the open interval $(-\infty, u)$.

(b) $\{(-|a|, |a| + 1)\}_{a \in \mathbb{Q}}$ stands for the function with domain $\mathbb{Q}$ and range, say, the set of all intervals, assigning each rational number a to the open interval $(-|a|, |a| + 1)$.

(c) Let p, q be real numbers.
For any real numbers a, b, not both zero, define $\ell_{(a,b)}$ by $\ell_{(a,b)} = \{(x, y) \mid x, y \in \mathbb{R} \text{ and } a(x - p) + b(y - q) = 0\}$.
$\ell_{(a,b)}$ is the line in the infinite plane whose equation is given by $ax + by = ap + bq$.
$\{\ell_{(a,b)}\}_{(a,b) \in \mathbb{R}^2 \backslash \{(0,0)\}}$ stands for the function with domain $\mathbb{R}^2 \backslash \{(0, 0)\}$ and range, say, the set of all lines in the infinite plane, assigning each $(a, b) \in \mathbb{R}^2 \backslash \{(0, 0)\}$ to the line $\ell_{(a,b)}$.
(The set of all terms of this family is the pencil of all lines passing through (p, q).)

(d) Let a, b, c be real numbers, not all zero.
For any real number d, define π_d by $\pi_d = \{(x, y, z) \mid x, y, z \in \mathbb{R} \text{ and } ax + by + cz = d\}$.
π_d is the plane in the infinite space whose equation is given by $ax + by + cz = d$.
$\{\pi_d\}_{d \in \mathbb{R}}$ stands for the function with domain $\mathbb{R}$ and range, say, the set of all planes in the infinite space, assigning each $d \in \mathbb{R}$ to the plane π_d.
(The set of all terms of this family is the pencil of planes all parallel to the plane $ax + by + cz = 0$.)

(e) For each complex number p, for each positive real number r, define $C(\zeta, r) = \{z \in \mathbb{C} : |z - p| = r\}$. ($C(\zeta, r)$ is the circle in the Argand plane with centre p and radius r.)
Let $p \in \mathbb{C}$.
$\{C(p, r)\}_{r \in (0,+\infty)}$ stands for the function with domain $(0, +\infty)$ and range, say, the set of all circles in the Argand plane, assigning each positive real number to the circle $C(p, r)$.
(The set of all terms of this family is the pencil of circles concentric at p.)

(f) For each complex number p, for each positive real number r, define $D(\zeta, r) = \{z \in \mathbb{C} : |z - p| < r\}$. ($D(\zeta, r)$ is the 'open disc' in the Argand plane with centre p and radius r.)

$\{D(r,r)\}_{r \in (0,+\infty)}$ stands for the function with domain $(0, +\infty)$ and range, say, the set of all open discs in the Argand plane, assigning each positive real number to the open disc $D(r,r)$.

(g) For each real number α, define the polynomial function $f_\alpha : \mathbb{R} \longrightarrow \mathbb{R}$ by $f_\alpha(x) = x^2 + \alpha x + 1$ for any $x \in \mathbb{R}$.

$\{f_\alpha\}_{\alpha \in \mathbb{R}}$ stands for the function with domain $\mathbb{R}$ and range, say, the set of all polynomial functions, assigning each real number α to f_α.

12. **Set operations for families of sets.**

The notion of intersection and union for infinite sequences of sets, introduced in the Handout *Universal quantifier and existential quantifier*, can be immediately generalized to families of sets.

Definition.

Let M, I be sets, and $\{S_\alpha\}_{\alpha \in I}$ be a family of subsets of the set M, indexed by I. (For any $\alpha \in I$, S_α is a subset of M.)

(1) *The* **intersection of the family of subsets** $\{S_\alpha\}_{\alpha \in I}$ **of the set** M *is defined to be the set $\{x \in M : x \in S_\alpha$ for any $\alpha \in I\}$. It is denoted by $\bigcap\limits_{\alpha \in I} S_\alpha$.*

(2) *The* **union of the family of subsets** $\{S_\alpha\}_{\alpha \in I}$ **of the set** M *is defined to be the set $\{x \in M : x \in S_\alpha$ for some $\alpha \in I\}$. It is denoted by $\bigcup\limits_{\alpha \in I} S_\alpha$.*

Remark. Suppose C is a subset of $\mathfrak{P}(M)$, and we 'index' C by its elements to obtain the family $\{S\}_{S \in C}$. Then the intersection of the set C of subsets of the set M is the intersection of the family $\{S\}_{S \in C}$, and the union of the set C of subsets of the set M is the union of the family $\{S\}_{S \in C}$.

13. Theorem $(\star)$ in the Handout *Universal quantifier and existential quantifier* can be generalized immediately to Theorem $(\star')$ below. The proofs of the respective statements are similar.

Theorem $(\star')$.

Let M, I be sets and $\{A_\alpha\}_{\alpha \in I}$ be a family of subsets of M, indexed by I.

(0) *Suppose $I = \emptyset$. Then $\bigcap\limits_{\alpha \in I} A_\alpha = M$ and $\bigcup\limits_{\alpha \in I} A_\alpha = \emptyset$.*

(1) *Let S be a subset of M. Suppose $S \subset A_\alpha$ for any $\alpha \in I$. Then $S \subset \bigcap\limits_{\alpha \in I} A_\alpha$.*

(2) *Let S be a subset of M. Suppose $S \subset A_\alpha$ for some $\alpha \in I$. Then $S \subset \bigcup\limits_{\alpha \in I} A_\alpha$.*

(3) *Let T be a subset of M. Suppose $A_\alpha \subset T$ for any $\alpha \in I$. Then $\bigcup\limits_{\alpha \in I} A_\alpha \subset T$.*

(4) *Let T be a subset of M. Suppose $A_\alpha \subset T$ for some $\alpha \in I$. Then $\bigcap\limits_{\alpha \in I} A_\alpha \subset T$.*

(5) *Let C be a subset of M. ($\{A_\alpha \cup C\}_{\alpha \in I}, \{A_\alpha \cap C\}_{\alpha \in I}, \{A_\alpha \backslash C\}_{\alpha \in I}, \{C \backslash A_\alpha\}_{\alpha \in I}$ are families of subsets of M.) The equalities below hold:*

(5a) $\left(\bigcap\limits_{\alpha \in I} A_\alpha\right) \cap C = \bigcap\limits_{\alpha \in I} (A_\alpha \cap C)$.

(5b) $\left(\bigcup\limits_{\alpha \in I} A_\alpha\right) \cup C = \bigcup\limits_{\alpha \in I} (A_\alpha \cup C)$.

(5c) $\left(\bigcap\limits_{\alpha \in I} A_\alpha\right) \cup C = \bigcap\limits_{\alpha \in I} (A_\alpha \cup C)$.

(5d) $\left(\bigcup\limits_{\alpha \in I} A_\alpha\right) \cap C = \bigcup\limits_{\alpha \in I} (A_\alpha \cap C)$.

(5e) $\left(\bigcap\limits_{\alpha \in I} A_\alpha\right) \backslash C = \bigcap\limits_{\alpha \in I} (A_\alpha \backslash C)$.

(5f) $\left(\bigcup\limits_{\alpha \in I} A_\alpha\right) \backslash C = \bigcup\limits_{\alpha \in I} (A_\alpha \backslash C)$.

(5g) $C \backslash \left(\bigcap\limits_{\alpha \in I} A_\alpha\right) = \bigcup\limits_{\alpha \in I} (C \backslash A_\alpha)$.

(5h) $C \backslash \left(\bigcup\limits_{\alpha \in I} A_\alpha\right) = \bigcap\limits_{\alpha \in I} (C \backslash A_\alpha)$.