MATH2060B Exercise 1

Deadline: Jan 13, 2015.

The questions are from Bartle and Sherbert, *Introduction to Real Analysis*, Wiley, 4th edition, unless otherwise stated.

Section 6.1 Q.5-10, 13, 14.

Section 6.2 Q.19.

Supplementary Exercises

1. A function $f : (a, b) \to \mathbb{R}$ has a symmetric derivative at $c \in (a, b)$ if

$$f_s'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c-h)}{2h}$$

   exists. Show that $f_s'(c) = f'(c)$ if the latter exists. But $f_s'(c)$ may exist even though f is not differentiable at c. Can you give an example?

2. Let $f : \mathbb{R} \to \mathbb{R}$ satisfy $f(x + y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$. Suppose f is differentiable at 0 with $f'(0) = 1$. Show that f is differentiable on $\mathbb{R}$ and $f'(x) = f(x)$ for all $x \in \mathbb{R}$.