HOMEWORK I (DEADLINE : 21ST SEPTEMBER, 2018)

ORDINARY DIFFERENTIAL EQUATIONS

Answer all questions:

(1) (4 points) **Solve** the following initial value problems (implicit solutions is also accepted):

(a) $t^4y' + 5t^3y = e^{-t}$, $y(-1) = 0$, for $t < 0$.

(b) $y' = y^2/t$, $y(1) = 3$.

(c) $y + (2t - 3ye^y)y' = 0$, $y(1) = 0$.

(d) $y' = ty^3(1 + t^2)^{-1/2}$, $y(0) = 1$.

(e) $y' = \frac{y-4t}{t-y}$, $y(1) = 3$, for $t > 0$.

(f) $y' = y - 2y^2$, $y(1) = 1$.

(g) $(3t^2y + 2ty + y^3) + (t^2 + y^2)y' = 0$, $y(0) = 1$.

(h) $(t^2 + 3ty + y^2) - t^2y' = 0$, $y(1) = 0$, for $t > 0$.

(2) (2 point) **Determine** whether each of the following equations is exact or not, if it is then **find** the solution:

(a) $(e^t \sin(y) - 3y\sin(t)) + (e^t \cos(y) + 3\cos(t))y' = 0$

(b) $(t + 2)\sin(y) + (t\cos(y))y' = 0$

(c) $\frac{t}{(t^2+y^2)^{3/2}} + \frac{y}{(t^2+y^2)^{3/2}}y' = 0$

(d) $y' = \frac{ay+b}{cy+d}$

(3) (2 points) Consider the general first order linear equation $y' = p(t)y + g(t)$, **show** that

- if $y_1(t)$ is a solution to $y' = p(t)y$, then $cy_1(t)$ is also a solution to $y' = p(t)y$ for $c \in \mathbb{R}$;

- if $y_2(t)$ is a solution to $y' = p(t)y + g(t)$, then $cy_1(t) + y_2(t)$ is also a solution to the equation $y' = p(t)y + g(t)$;

- all the solutions to $y' = p(t)y + g(t)$ is of the form $cy_1(t) + y_2(t)$ for some $c \in \mathbb{R}$.

(4) (2 points) Consider the differential equation

$$M(t, y) + N(t, y)y' = 0. \tag{0.1}$$

Assume that we have $tM - yN \neq 0$, and the fraction $(\frac{dN}{dt} - \frac{dM}{dy})/(tM - yN) = R(ty)$ depending only on the quantity ty only, then **show** that the differential equation 0.1 has a integrating factor of the form $\mu(ty)$ and **find** a general formula for this integrating factor.

End of Homework 1