MATH4060 Exercise 2

Due Date: October 4, 2016.

The questions are from Stein and Shakarchi, *Complex Analysis*, unless otherwise stated.

Chapter 3. Exercise 12.

Chapter 4. Exercise 1, 2, 3, 6, 7.

Chapter 5. Exercise 2, 3, 6, 7, 8, 9.

Additional Exercise. Show that if $0 < a < 1$, then

$$\int_0^\infty \frac{v^{a-1}}{1+v} dv = \frac{\pi}{\sin \pi a}.$$

(Hint: Consider, for a small $\delta > 0$, the contour $\gamma_1 + \gamma_2 - \gamma_3 - \gamma_4$, where

$$\begin{aligned}
\gamma_1 &= \{t + i\delta \colon \varepsilon \le t \le R\}, \\
\gamma_2 &= \{(R^2 + \delta^2)^{1/2} e^{i\theta} \colon \arctan(\delta/R) \le \theta \le 2\pi - \arctan(\delta/R)\}, \\
\gamma_3 &= \{t - i\delta \colon \varepsilon \le t \le R\}, \\
\gamma_4 &= \{(\varepsilon^2 + \delta^2)^{1/2} e^{i\theta} \colon \arctan(\delta/\varepsilon) \le \theta \le 2\pi - \arctan(\delta/\varepsilon)\},
\end{aligned}$$

and consider a suitable branch of $v \mapsto v^{a-1}$ for v away from the non-negative real axis.)

Correction. In Chapter 5, Exercise 7(a), one should add the condition that $a_n \neq -1$ for all n.