THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH1010 University Mathematics (Fall 2020)
Suggested Solution of Coursework 3

If you find any errors or typos, please email us at
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1. (1 point) Given $\lim_{x\to 8} f(x) = 5$ and $\lim_{x\to 8} g(x) = 3$, evaluate

$$\lim_{x\to 8} \frac{f(x)}{g(x)}.$$

If the limit does not exist enter DNE.
Limit = ____________

Solution:

$$\begin{aligned}\lim_{x\to 8} \frac{f(x)}{g(x)} &= \frac{\lim\limits_{x\to 8} f(x)}{\lim\limits_{x\to 8} g(x)}\\ &= \frac{5}{3}\end{aligned}$$

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2. (1 point) Using: $\lim\limits_{x\to 6} f(x) = 5$ and $\lim\limits_{x\to 6} g(x) = 3$, evaluate

$$\lim_{x\to 6} \frac{f(x)+g(x)}{6f(x)}.$$

Limit = ____
Enter **DNE** if the limit does not exist.

Solution:

$$\begin{aligned}\lim_{x\to 6} \frac{f(x)+g(x)}{6f(x)} &= \frac{\lim\limits_{x\to 6}(f(x)+g(x))}{6\lim\limits_{x\to 6} f(x)}\\ &= \frac{4}{15} \approx 0.266667\end{aligned}$$

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3. (1 point) Evaluate the limit

$$\lim_{x\to -1} \frac{4x^2-4x+5}{x-7}$$

If the limit does not exist enter DNE.
Limit = ____________

Solution:

$$\begin{aligned}\lim_{x\to -1} \frac{4x^2-4x+5}{x-7} &= \frac{\lim\limits_{x\to -1}(4x^2-4x+5)}{\lim\limits_{x\to -1}(x-7)}\\ &= -\frac{13}{8}\end{aligned}$$

4. (1 point) Evaluate the limit

$$\lim_{x\to 6}\left(\sqrt{x^2+2}-\frac{x^2+6x}{x}\right)$$

If the limit does not exist enter DNE.

Limit = ____________

Solution:

$$\begin{aligned}\lim_{x\to 6}\left(\sqrt{x^2+2}-\frac{x^2+6x}{x}\right) &= \lim_{x\to 6}\sqrt{x^2+2}-\lim_{x\to 6}\frac{x^2+6x}{x}\\ &= \sqrt{38}-12\\ &\approx -5.83558599703102\end{aligned}$$

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5. (1 point) Evaluate the limit

$$\lim_{x\to 0} 5\ln x.$$

Enter **DNE** if the limit does not exist.

Limit = _____

Solution:

The limit does not exist.

As x approaches 0 from the right, $5\ln x$ becomes unbounded in the negative sense.

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6. (1 point) Let f be defined by

$$f(x)=\begin{cases}4x^3-2m, & x\le -1\\ 8x^2+5m, & x>-1\end{cases}$$

(a) Find (in terms of m) $\lim\limits_{x\to -1^+} f(x)$

Limit = _____

(b) Find (in terms of m) $\lim\limits_{x\to -1^-} f(x)$

Limit = _____

(c) Find the value of m so that

$$\lim_{x\to -1^-} f(x)=\lim_{x\to -1^+} f(x)$$

m = _______

Solution:

(a)

$$\begin{aligned}\lim_{x\to -1^+} f(x) &= \lim_{x\to -1^+}(8x^2+5m)\\ &= 8+5m\end{aligned}$$

(b)

$$\begin{aligned}\lim_{x\to -1^-} f(x) &= \lim_{x\to -1^-}(4x^3-2m)\\ &= -4-2m\end{aligned}$$

(c)

$$\begin{aligned}8+5m &= -4-2m\\ m &= -\frac{12}{7}\\ m &\approx -1.71428571428571\end{aligned}$$

7. (1 point)

Let $f(x) = \begin{cases} \sqrt{-2-x}+2, & \text{if } x < -3 \\ 2, & \text{if } x = -3 \\ 3x+12, & \text{if } x > -3 \end{cases}$

Calculate the following limits. Enter **DNE** if the limit does not exist.

$\lim_{x\to -3^-} f(x) =$ _______

$\lim_{x\to -3^+} f(x) =$ _______

$\lim_{x\to -3} f(x) =$ _______

Solution: (a)

$$\begin{aligned}\lim_{x\to -3^-} f(x) &= \lim_{x\to -3^-} (\sqrt{-2-x}+2) \\ &= 3\end{aligned}$$

(b)

$$\begin{aligned}\lim_{x\to -3^+} f(x) &= = \lim_{x\to -3^+} (3x+12) \\ &= 3\end{aligned}$$

(c)

Because $\lim_{x\to -3^+} f(x) = \lim_{x\to -3^-} f(x)$, $\lim_{x\to -3} f(x)$ exists.

$$\lim_{x\to -3} f(x) = \lim_{x\to -3^+} f(x) = \lim_{x\to -3^-} f(x) = 3.$$

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8. (1 point)

Is it possible for $\lim_{x\to 1} f(x)$ to exist when $\lim_{x\to 1^-} f(x) = 3$ and $\lim_{x\to 1^+} f(x) = 7$?

Choose the answer from the Drop-down menu.

Answer : [Yes/No]

Solution: The limit $\lim_{x\to 1} f(x)$ exists when $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$, so the limit does not exist.

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9. (1 point) Evaluate the limits.

$$g(x) = \begin{cases} 2x+2 & x < -7 \\ -14 & x = -7 \\ 2x-2 & x > -7 \end{cases}$$

Enter **DNE** if the limit does not exist.

a) $\lim_{x\to -7^-} g(x) =$ _____

b) $\lim_{x\to -7^+} g(x) =$ _____

c) $\lim_{x\to -7} g(x) =$ _____

d) $g(-7) =$ _____

Solution:

(a)

$$\begin{aligned}\lim_{x\to -7^-} g(x) &= \lim_{x\to -7^-} (2x+2) \\ &= -12\end{aligned}$$

(b)

$$\begin{aligned}\lim_{x\to -7^+} g(x) &= \lim_{x\to -7^+}(2x-2)\\ &= -16\end{aligned}$$

(c)

$$\lim_{x\to -7^+} g(x) \neq \lim_{x\to -7^-} g(x)$$

So the limit does not exist.
(d)

$$g(-7) = -14$$

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10. (1 point) Determine the following limits. If a limit *does not exist,* type **DNE**.

$$f(x) = \begin{cases} x-2, & \text{for } x \leq -1 \\ x^2+3, & \text{for } -1 < x \leq 1 \\ 5-x, & \text{for } x > 1 \end{cases}$$

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1. $\lim_{x\to -1^-} f(x) =$ __________
2. $\lim_{x\to -1^+} f(x) =$ __________
3. $\lim_{x\to -1} f(x) =$ __________
4. $\lim_{x\to 1} f(x) =$ __________
5. $f(-1) =$ __________

Solution:
(1)

$$\begin{aligned}\lim_{x\to -1^-} f(x) &= \lim_{x\to -1^-}(x-2)\\ &= -3\end{aligned}$$

(2)

$$\begin{aligned}\lim_{x\to -1^+} f(x) &= \lim_{x\to -1^+}(x^2+3)\\ &= 4\end{aligned}$$

(3) $\lim_{x\to -1} f(x)$ does not exist.
(4)

$$\begin{aligned}\lim_{x\to 1^+} f(x) &= \lim_{x\to 1^+}(5-x)\\ &= 4\end{aligned}$$

$$\begin{aligned}\lim_{x\to 1^-} f(x) &= \lim_{x\to 1^-}(x^2+3)\\ &= 4\end{aligned}$$

$$\lim_{x\to 1} f(x) = \lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x) = 4$$

(5)

$$f(-1) = -1-2 = -3$$

11. (1 point) Evaluate the limit

$$\lim_{x\to\frac{2}{17}} \frac{17x^2-2x}{|17x-2|}$$

Enter **INF** for ∞, **-INF** for $-\infty$, and **DNE** if the limit does not exist.

Limit = ____________

Solution:

$$\begin{aligned}\lim_{x\to\frac{2}{17}^+} \frac{17x^2-2x}{|17x-2|} &= \lim_{x\to\frac{2}{17}^+} \frac{17x^2-2x}{17x-2}\\ &= \lim_{x\to\frac{2}{17}^+} x\\ &= \frac{2}{17}\\ \lim_{x\to\frac{2}{17}^-} \frac{17x^2-2x}{|17x-2|} &= \lim_{x\to\frac{2}{17}^-} \frac{17x^2-2x}{2-17x}\\ &= \lim_{x\to\frac{2}{17}^-} (-x)\\ &= -\frac{2}{17}\\ \lim_{x\to\frac{2}{17}^+} \frac{17x^2-2x}{|17x-2|} &\neq \lim_{x\to\frac{2}{17}^-} \frac{17x^2-2x}{|17x-2|}\end{aligned}$$

So the limit does not exist.

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12. (1 point)

Evaluate the limit, if it exists. If not, enter *DNE* below.

$$\lim_{t\to-4} \frac{t^2-16}{-3t^2-8t+16}$$

Answer = _____

Solution:

$$\begin{aligned}\lim_{t\to-4} \frac{t^2-16}{-3t^2-8t+16} &= \lim_{t\to-4} \frac{(t+4)(t-4)}{(t+4)(-3t+4)}\\ &= \lim_{t\to-4} \frac{t-4}{-3t+4}\\ &= -\frac{1}{2}\end{aligned}$$

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13. (1 point) Consider the following limit

$$\lim_{x\to 8} \frac{72-8x-|x^2-9x|}{|x^2-81|-17}$$

We can simplify this limit by rewriting it as an expression without absolute values as follows

$\lim_{x\to 8}$ ____________________

We can then cancel off a common factor in the numerator and denominator, thus simplifying our limit to

$\lim_{x\to 8}$ ________________

We can then evaluate the limit directly and find that its value is _________

Solution:
(1) $x^2 - 9x \le 0$, and $x^2 - 81 \le 0$.

$$\begin{aligned}\lim_{x\to 8} \frac{72-8x-|x^2-9x|}{|x^2-81|-17} &= \lim_{x\to 8} \frac{72-8x+x^2-9x}{81-x^2-17} \\ &= \lim_{x\to 8} \frac{72-17x+x^2}{64-x^2}\end{aligned}$$

(2)

$$\lim_{x\to 8} \frac{72-17x+x^2}{64-x^2} = \lim_{x\to 8} \frac{(x-8)(x-9)}{-(x-8)(x+8)} = \lim_{x\to 8} \frac{9-x}{x+8}$$

(3)

$$\lim_{x\to 8} \frac{9-x}{x+8} = \frac{1}{16}$$

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14. (1 point)
Evaluate the limit:
$\lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{4(x-a)} =$ ___

Solution:

$$\begin{aligned}\lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{4(x-a)} &= \lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{4(\sqrt{x}-\sqrt{a})(\sqrt{x}+\sqrt{a})} \\ &= \lim_{x\to a} \frac{1}{4(\sqrt{x}+\sqrt{a})} \\ &= \frac{1}{8\sqrt{a}}\end{aligned}$$

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15. (1 point) Evaluate the following limits:
$\lim_{x\to -2} \frac{x^2-4}{x^2+6x+8} =$ ________
$\lim_{x\to -\infty} \frac{x^2-4}{x^2+6x+8} =$ ________
$\lim_{x\to -5} \frac{\sqrt{3x^2-50}-5}{x+5} =$ ________
Solution:
(1)

$$\begin{aligned}\lim_{x\to -2} \frac{x^2-4}{x^2+6x+8} &= \lim_{x\to -2} \frac{(x+2)(x-2)}{(x+2)(x+4)} \\ &= \lim_{x\to -2} \frac{x-2}{x+4} \\ &= -2\end{aligned}$$

(2)

$$\begin{aligned}\lim_{x\to -\infty} \frac{x^2-4}{x^2+6x+8} &= \lim_{x\to -\infty} \frac{(x+2)(x-2)}{(x+2)(x+4)} \\ &= \lim_{x\to -\infty} \frac{x-2}{x+4} \\ &= \lim_{x\to -\infty} (1-\frac{6}{x+4}) \\ &= 1\end{aligned}$$

(3)

$$\begin{aligned}\lim_{x\to -5}\frac{\sqrt{3x^2-50}-5}{x+5} &= \lim_{x\to -5}\frac{(\sqrt{3x^2-50}-5)(\sqrt{3x^2-50}+5)}{(x+5)(\sqrt{3x^2-50}+5)}\\ &= \lim_{x\to -5}\frac{3(x+5)(x-5)}{(x+5)(\sqrt{3x^2-50}+5)}\\ &= \lim_{x\to -5}\frac{3(x-5)}{(\sqrt{3x^2-50}+5)}\\ &= -3\end{aligned}$$

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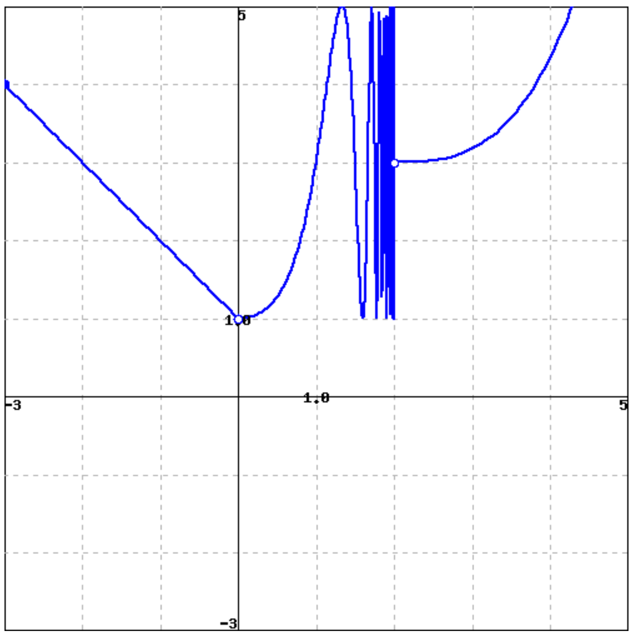
17. (1 point) Use the given graph of the function f to find the following limits. If a limit *does not exist,* type "DNE".



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1. $\lim_{x\to 2^-} f(x) =$ ________
2. $\lim_{x\to 2^+} f(x) =$ ________
3. $\lim_{x\to 2} f(x) =$ ________
4. $\lim_{x\to 0} f(x) =$ ________
5. $f(0) =$ ________

Note: *You can click on the graph to enlarge the image.*

Solution:
(1)$\lim_{x\to 2^-} f(x)$ does not exist.
(2)$\lim_{x\to 2^+} f(x) = 3$
(3)$\lim_{x\to 2} f(x)$ does not exist.
(4)$\lim_{x\to 0} f(x) = 1$
(5)$f(0)$ does not exist.

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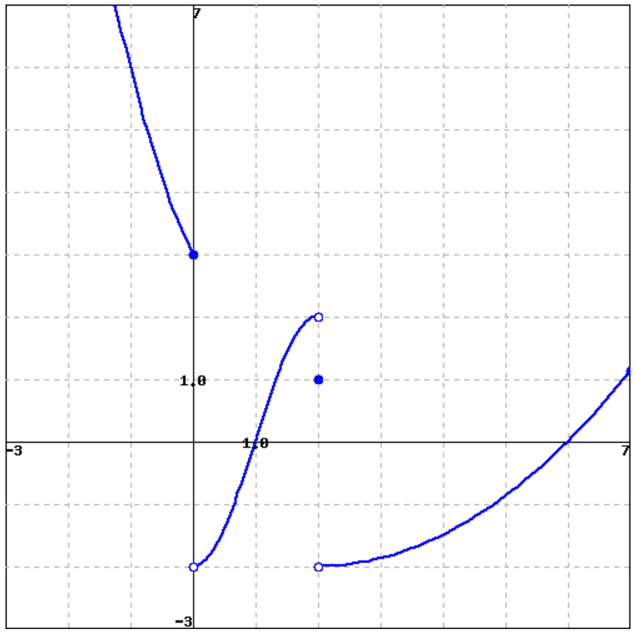
18. (1 point) Use the given graph of the function g to find the following limits:



1. $\lim_{x \to 2^-} g(x) =$ ________
2. $\lim_{x \to 2^+} g(x) =$ ________
3. $\lim_{x \to 2} g(x) =$ ________
4. $\lim_{x \to 0} g(x) =$ ________
5. $g(2) =$ ________

Note: *You can click on the graph to enlarge the image.*

Solution:
(1)$\lim_{x \to 2^-} g(x) = 2$
(2)$\lim_{x \to 2^+} g(x) = -2$
(3)Because $\lim_{x \to 2^-} g(x) \neq \lim_{x \to 2^+} g(x)$, $\lim_{x \to 2} g(x)$ does not exist.
(4)Because $\lim_{x \to 0^-} g(x) \neq \lim_{x \to 0^+} g(x)$, $\lim_{x \to 0} g(x)$ does not exist.
(5)$g(2) = 1$

19. (1 point) The function

$$f(x) = \frac{2x^2 - 5x + 3}{x - 2}$$

has a vertical asymptote at $x =$ _____
and an oblique asymptote with the equation

$$y = mx + b$$

where $m =$ _____ and $b =$
_____. (You can use synthetic or long division to compute the equation of that asymptote.)
Solution:
(1)$f(x) = 2x - 1 + \frac{1}{x-2}$,

$$\lim_{x \to 2^-} f(x) = -\infty$$

$$\lim_{x \to 2^+} f(x) = +\infty$$

So the function has a vertical asymptote at $x = 2$.
(2)$f(x) = 2x - 1 + \frac{1}{x-2}$, $m = 2$ and $b = -1$.

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20. (1 point) Let $f(x)$ be a function such that

$$\lim_{x\to\infty} f(x) = \infty \quad \lim_{x\to-\infty} f(x) = 10$$

$$\lim_{x\to 3^+} f(x) = \infty \quad \lim_{x\to 3^-} f(x) = -\infty$$

Determine the horizontal asymptote.

$y =$ _____

Determine the vertical asymptote.
$x =$ _____

Solution:
(1)$\lim_{x\to-\infty} f(x) = 10$, so the horizontal asymptote is $y = 10$.
(2)$\lim_{x\to 3^+} f(x) = \infty$ and $\lim_{x\to 3^-} f(x) = -\infty$, so the vertical asymptote is $x = 3$.

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21. (1 point) Let

$$f(x) = \frac{x}{\sqrt[4]{x^4+2}}.$$

Find the horizontal and vertical asymptotes of $f(x)$. If there are no asymptotes of a given type, enter 'none'. If there are more than one of a given type, list them separated by commas.

Horizontal asymptote(s): $y =$ __________

Vertical asymptote(s): $x =$ __________

Solution:
(1)

$$\lim_{x\to-\infty} f(x) = -1$$

and

$$\lim_{x\to\infty} f(x) = 1$$

So the horizontal asymptotes are $y = -1$ and $y = 1$.
(2) There exists no x_0 such that

$$\lim_{x\to {x_0}^-/{x_0}^+} f(x) = -\infty/+\infty.$$

Thus, the vertical asymptote does not exist.