THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH1010 University Mathematics (Fall 2020)
Suggested Solution of Coursework 2

(1) The function $h(x)$ is continuous at every number in its domain. State domain.

$$h(x) = \frac{\sin(4x)}{x+4}$$

Solutions:
Since $x+4$ is on the denominator, $x+4 \neq 0$, which implies $x \neq -4$. For any $x \neq -4$, $\sin(4x)$ is continuous, $x+4$ is continuous and nonzero. Hence, the domain of $h(x)$ is $(-\infty, -4) \cup (-4, \infty)$.

(2) Find the domain of the function

$$f(x) = \frac{\sqrt{10-3x}}{x^2-25}$$

Solutions:
Since we take the square root of $10-3x$, $10-3x \geq 0$, which implies $x \leq \frac{10}{3}$. Since x^2-25 is on the denominator, $x^2-25 \neq 0$, which implies $x \neq \pm 5$. Hence, the domain of $f(x)$ is $(-\infty, -5) \cup (-5, \frac{10}{3}]$.

(3) Use interval notation to indicate the domain of

$$f(x) = \sqrt[4]{x^2-7x}$$

and

$$g(x) = \sqrt[3]{15x^2-6x}.$$

Solutions:
Since we take the quartic root of x^2-7x, $x^2-7x \geq 0$, which implies $x \leq 0$ or $x \geq 7$. Hence, the domain of $f(x)$ is $(-\infty, 0] \cup [7, \infty)$.

Since taking the cubic root has no restriction on the domain and $15x^2-6x$ is well defined on the real line, the domain of $g(x)$ is $(-\infty, \infty)$.

(4) Find the domain of the function

$$g(x) = \log_a(x^2-16).$$

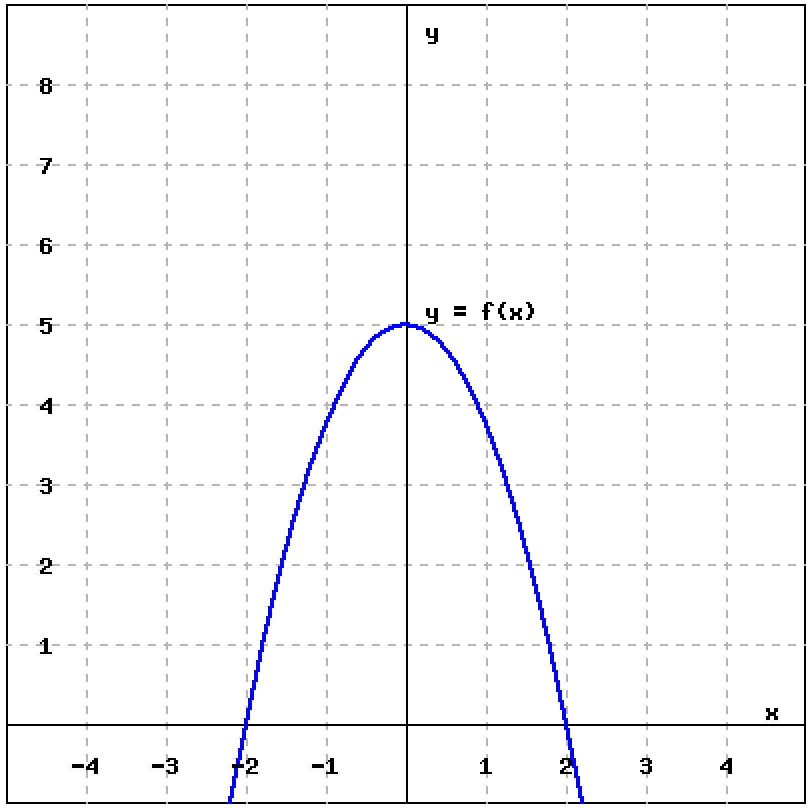
Solutions:
Since we take the logarithm of x^2-16, $x^2-16 > 0$, which implies $x < -4$ or $x > 4$. Hence, the domain of $g(x)$ is $(-\infty, -4) \cup (4, \infty)$.

(5) Given that $f(x) = \frac{1}{x}$ and $g(x) = 5x+4$, calculate $f \circ g(x)$, $g \circ f(x)$, $f \circ f(x)$, $g \circ g(x)$ and find their domains.

Solutions:
$f \circ g(x) = f(g(x)) = f(5x+4) = \frac{1}{5x+4}$.
The domain of $f \circ g(x)$ is $(-\infty, -\frac{4}{5}) \cup (-\frac{4}{5}, \infty)$.
$g \circ f(x) = g(f(x)) = g(\frac{1}{x}) = \frac{5}{x} + 4$.
The domain of $g \circ f(x)$ is $(-\infty, 0) \cup (0, \infty)$.
$f \circ f(x) = f(f(x)) = \frac{1}{\frac{1}{x}}$.
The domain of $f \circ f(x)$ is $(-\infty, 0) \cup (0, \infty)$.
(Remark: $f \circ f(x) = x$ on $(-\infty, 0) \cup (0, \infty)$, but it is not well defined at $x = 0$)
$g \circ g(x) = g(g(x)) = 5(5x+4) + 4 = 25x + 24$.
The domain of $g \circ g(x)$ is $(-\infty, \infty)$.

(6) Given the functions $f(x) = \frac{x-4}{x-6}$ and $g(x) = \sqrt{x+3}$, find the domains of f, g, $f+g$, $\frac{f}{g}$, $\frac{g}{f}$, $f \circ g$, $g \circ f$.
Solutions:
The domain of f is $(-\infty, 6) \cup (6, \infty)$.
The domain of g is $[-3, \infty)$.
$(f+g)(x) = \frac{x-4}{x-6} + \sqrt{x+3}$.
The domain of $f+g$ is $[-3, 6) \cup (6, \infty)$.
$\frac{f}{g}(x) = \frac{x-4}{(x-6)\sqrt{x+3}}$.
The domain of $\frac{f}{g}$ is $(-3, 6) \cup (6, \infty)$.
$\frac{g}{f}(x) = \frac{\sqrt{x+3}}{\frac{x-4}{x-6}}$.
The domain of $\frac{g}{f}$ is $[-3, 4) \cup (4, 6) \cup (6, \infty)$.
$f \circ g(x) = f(g(x)) = f(\sqrt{x+3}) = \frac{\sqrt{x+3}-4}{\sqrt{x+3}-6}$.
Since we take the square root of $x+3$, $x \geq -3$. Since $\sqrt{x+3} - 6$ is on the denominator, $\sqrt{x+3} - 6 \neq 0$,which implies $x \neq 33$.
Hence, the domain of $f \circ g$ is $[-3, 33) \cup (33, \infty)$.
$g \circ f(x) = g(f(x)) = \sqrt{\frac{x-4}{x-6} + 3}$.
Since $x - 6$ is on the denominator, $x \neq 6$. Since we take square root of $\frac{x-4}{x-6} + 3$, $\frac{x-4}{x-6} + 3 \geq 0$, which implies $x > 6$ or $x \leq \frac{11}{2}$. Hence, the domain of $g \circ f$ is $(-\infty, \frac{11}{2}] \cup (6, \infty)$.

(7) Use the graph to find the missing values



Solutions:
$f(0) = 5$
$f(\pm 2) = 0.$

(8) Express the function $y = 4x^4 - 2$ as a composition $y = f(g(x))$ of two simpler functions $y = f(u)$ and $u = g(x)$.
Solutions:
Actually, the problem has many different solutions. It depends on how you understand the word 'simpler'. I will give one of them.
$f(u) = 4u - 2$
$g(x) = x^4.$

(9) Use substitution to compose $D = 9p - 6$ and $p = 4q^5$. Enter your answer as an equation, and simplify your answer as much as possible.
Solutions:
$D = 9p - 6 = 9 \times 4q^5 - 6 = 36q^5 - 6.$

(10) Suppose $f(x) = 6x - 9$ and $g(y) = \frac{y}{6} + \frac{9}{6}$.

(a) Find the composition $g(f(x))$.

(b) Find the composition $f(g(y))$.

(c) Are the functions f and g inverse to each other?

Solutions:

(a) $g(f(x)) = g(6x - 9) = \frac{6x-9}{6} + \frac{9}{6} = x$.

(b) $f(g(y)) = f(\frac{y}{6} + \frac{9}{6}) = 6 \times \frac{y}{6} + \frac{9}{6} - 9 = y$.

(c) Yes.

(11) Find the inverse function to $y = f(x) = 3x + 7$.
Solutions:
Expressing x in terms of y, we have $x = \frac{y}{3} - \frac{7}{3}$. Hence, $x = g(y) = \frac{y}{3} - \frac{7}{3}$.

(12) Find the inverse function to $y = f(x) = \frac{8-6x}{5-4x}$.
Solutions:
We express x in terms of y.

$$\begin{aligned} y(5 - 4x) &= 8 - 6x \\ 5y - 4xy &= 8 - 6x \\ 5y - 8 &= x(4y - 6) \\ x &= \frac{8 - 5y}{6 - 4y} \end{aligned}$$

Hence, $x = g(y) = \frac{8-5y}{6-4y}$.

(13) Find the domain, x-intercept(s), y-intercept(s), and symmetry of the function

$$f(x) = 9 - x^2.$$

Solutions:

(a) The domain of f is $(-\infty, \infty)$.

(b) We solve the equation

$$\begin{aligned} f(x) &= 0 \\ 9 - x^2 &= 0 \\ x &= \pm 3 \end{aligned}$$

Hence, the x-intercepts of f are $(-3, 0)$ and $(3, 0)$.

(c) Taking $x = 0$, we have $f(x) = 9$.
Hence, the y-intercept of f is $(0, 9)$.

(d) Since $f(0) \neq 0$, f is not odd.
Since $f(-x) = 9 - (-x)^2 = 9 - x^2 = f(x)$, f is even.

(14) Find the domain, x-intercept(s), y-intercept(s), and symmetry of the function

$$f(x) = x(x - 11)(x + 8).$$

Solutions:

(a) The domain of f is $(-\infty, \infty)$.

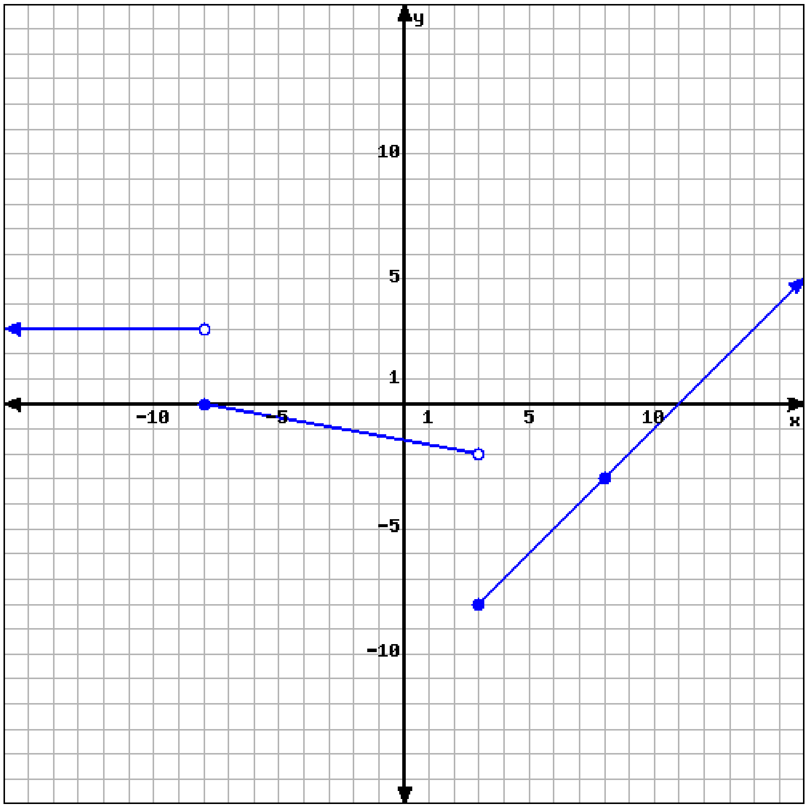
(b) We solve the equation

$$f(x) = 0$$
$$x(x - 11)(x + 8) = 0$$
$$x = 0, 11, -8$$

Hence, the x-intercepts of f are $(0, 0)$,$(11, 0)$,$(-8, 0)$.

(c) Taking $x = 0$, we have $f(x) = 0$.
Hence, the y-intercept of f is $(0, 0)$.

(d) Since $f(-8) = 0$ and $f(8) = -384$, f is neither odd nor even.



(15) The graph is a piecewise function, $f(x)$, is depicted above. Find its equation.
Solutions:
For $x < -8$, the graph of f is constant 3.

For $-8 \leq x < 3$, f is linear and $f(-8) = 0$, $f(3^-) = -2$. Assume $f(x) = ax + b$.
We solve the linear system

$$\begin{cases} -8a + b = 0 \\ 3a + b = -2 \end{cases}$$

We have

$$\begin{cases} a = -\frac{2}{11} \\ b = -\frac{16}{11} \end{cases}$$

For $x \geq 3$,f is linear and $f(3) = -8$, $f(8) = -3$. Assume $f(x) = ax + b$.
We solve the linear system

$$\begin{cases} 3a + b = -8 \\ 8a + b = -3 \end{cases}$$

We have

$$\begin{cases} a = 1 \\ b = -11 \end{cases}$$

Hence, we have

$$f(x) = \begin{cases} 3, & x < -8, \\ -\frac{2}{11}x - \frac{16}{11}, & -8 \le x < 3, \\ x - 11, & x \ge 3. \end{cases}$$

(16) Choose the graph that represents the piecewise function

$$\begin{cases} \frac{1}{2}x - 4, & x \le -1, \\ -\frac{1}{5}x - 3, & x > -1. \end{cases}$$

from the below choices.

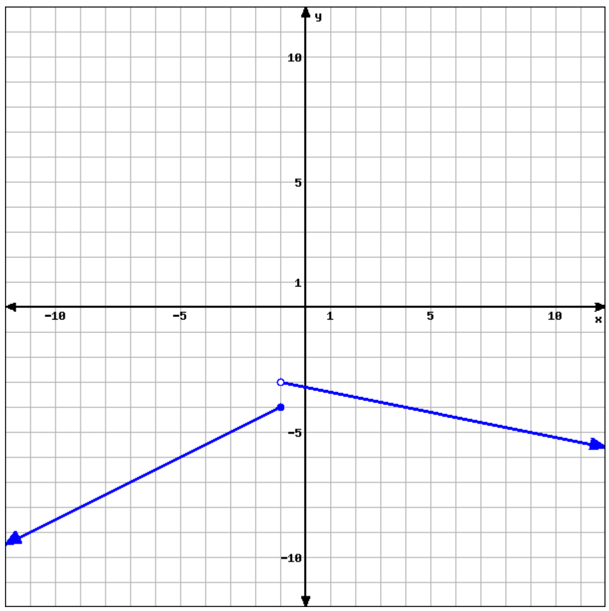


FIGURE (1) A

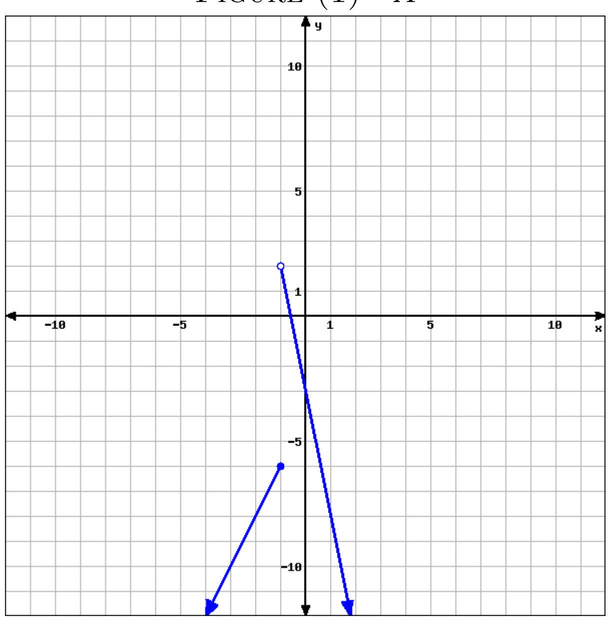


FIGURE (2) B

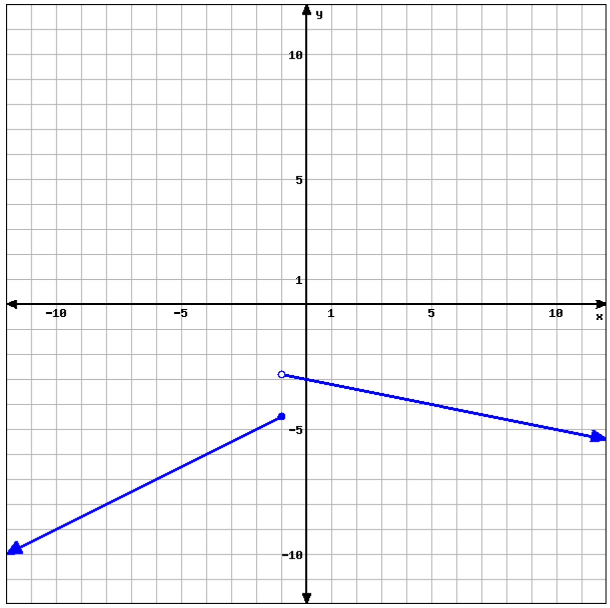


FIGURE (3) C

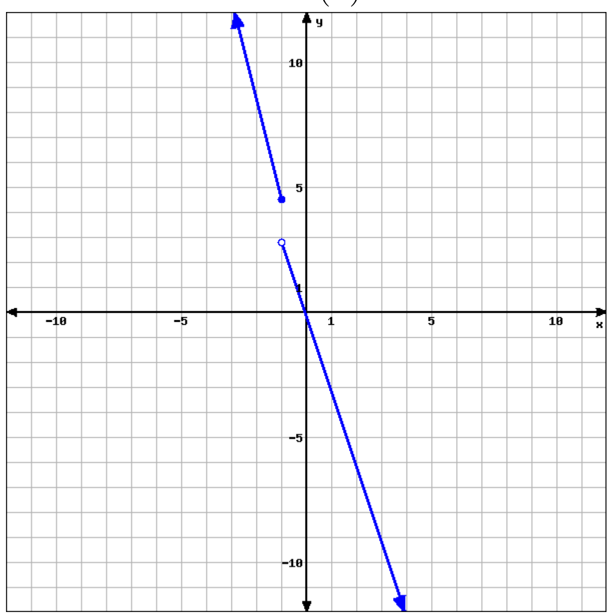


FIGURE (4) D

Solutions:
Note that $f(-1) = -\frac{9}{2}$ and $f(-1^+) = -\frac{14}{3}$. The only graph satisfying this is C.

(17) A local pet shop charges \$0.70 per cricket up to 170 crickets, and \$0.61 per cricket thereafter. Write a piecewise-defined linear function which calculates the price P, in dollars, of purchasing c crickets.
Solutions:
Note that we have $P(0) = 0$, the slope of the first section is 0.7 and that of the second section is 0.61.
Also, for the second section, the starting point is $(170, P(170)) = (170, 119)$. Assume the formula is $P(c) = 0.61c + b$. We have $119 = 0.61 \times 170 + b$, which implies

$b = 15.3$.
Hence, P(c) =

$$\begin{cases} 0.7c, & 1 \leq c \leq 170, \\ 0.61c + 15.3, & c > 170. \end{cases}$$