How is University mathematics different from high school?

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Impressions about high school mathematics

- Broadly divided into calculus, algebra and geometry
- Main emphasis: knowing how to calculate, and obtaining the 'correct answer'

How is University mathematics different from high school?

- Breadth
- Depth
- Focus

Breadth

- We still have calculus, algebra and geometry at the University.
- But they come in very different flavors:
  - Multivariable calculus, real analysis, complex analysis
  - study of symmetries, groups, rings, fields, commutative algebra
  - Algebraic geometry, Riemannian geometry, sympletic geometry, complex geometry, conformal geometry, contact geometry ...
- On top of that, also many areas of mathematics that may be new to you:
  - functional analysis, harmonic analysis, convex analysis;
  - partial differential equations, mathematical physics;
  - numerical analysis, operations research, image processing, optimization, logistics, financial mathematics, game theory;
  - analytic number theory, algebraic number theory;
  - representation theory, supersymmetry;
  - stochastic processes, ergodic theory, dynamical systems;
  - differential topology, algebraic topology, low dimensional topology, knot theory ...

Depth

- Not just about isolated theorems, but also about theories: e.g. Galois theory, measure theory, gauge theory, KAM theory . . .
- The deepest theorems often reveal connections between different areas of mathematics.
- Ultimately university mathematics will lead one to frontiers of current research (territories completely open and unknown)

Focus

- More emphasis on conceptual understandings than calculations (hence more abstract)
- More emphasis on definitions, theorems, proofs
- A higher demand for precise statements, logical deductions, and concise presentations
- More importantly, one learns to distinguish
  - true statements from false ones, and
  - logical reasonings from illogical ones
- More open-ended; hence creativity and imagination also play a much bigger role

What effect does this have about learning University mathematics?

- Memorization is very inefficient, and also less fun
- Needs to gain real understandings instead
- Needs to be critical about information you receive (even if that comes from a trusted source, or from yourself)
- Simple concrete (even extreme) examples often help, when it comes to understanding abstract definitions and theories
- Language skills become more important (in conveying an idea, or giving a clear presentation)
- Needs to be brave about making mistakes
  (Mistakes are fine as long as you can catch them; deliberate mistakes are often even desirable in creative processes, because they help eliminate the impossibilities, and help one zero in on the correct way to proceed)

Three stages of learning more advanced mathematics

- Initially one learns about calculations (1st-2nd year college)
- Gradually one shifts to the foundations of the subject, carefully establishing results taken for granted before (2nd-3rd year)
- Then one goes into more advanced topics, specializing in various areas of interest (3rd year onwards)

How can one learn mathematics well at a more advanced level?

- Learn to distinguish assumptions from conclusions
- Learn to test assumptions of theorems
  (and see whether the theorems apply)
- Learn through examples and exercises
  (*Math is not a spectator sport!* Hard work is rewarded)
- Learn from your friends (Math is very collaborative nowadays)
- Come up with examples and exercises of your own
- Discover your own proofs
- Be curious
- Ask questions! Even "dumb" ones. And answer them.
- Don't be afraid of challenging conventional wisdom
- Enjoy mathematics!