Math 1010C Term 1 2014
Supplementary exercises 2

The following exercises are not to be submitted, but they form an important part of the course, and you're advised to go through them carefully.

1. Solve Exercises 1-14 of Section 4.1 of Thomas' calculus (11th edition).

2. Let $f(x) = x^3 - 6x^2 + 9x - 2$.
   (a) Show that f is continuous on $\mathbb{R}$.
   (b) What does the extreme value theorem tell us about f on the interval $[-1, 2]$? (State carefully what you use about the interval $[-1, 2]$ when you apply the extreme value theorem.)
   (c) We know by part (b) that there exists (at least one) $x_0 \in [-1, 2]$ such that f achieves its (absolute) maximum on $[-1, 2]$ at x_0. Show that if x_0 is not one of the boundary points, i.e. if $x_0 \neq -1$ nor 2, then x_0 is a critical point of f.
   (d) Hence show that x_0 cannot be anything other than -1, 2, or 1.
   (e) Show that actually x_0 cannot be -1 or 2.
   (f) What must x_0 be then?
   (g) Using the above, find the maximum value of f on $[-1, 2]$.
   (h) We are going to repeat the above for the (absolute) minimum of f on $[-1, 2]$. We know by part (b) that there exists (at least one) $y_0 \in [-1, 2]$ such that f achieves its (absolute) minimum on $[-1, 2]$ at y_0. Show that if y_0 is not one of the boundary points, i.e. if $y_0 \neq -1$ nor 2, then y_0 is a critical point of f.
   (i) Hence show that y_0 cannot be anything other than -1, 2, or 1.
   (j) Show that actually y_0 cannot be 1 or 2.
   (k) What must y_0 be then?
   (l) Using the above, find the minimum value of f on $[-1, 2]$.
   (m) Confirm your answer by plotting a graph of f, say using a graphing calculator or a computer software.

3. Let f be as in Question 2.
   (a) Repeat the above question, to find the maximum and minimum value of f on the interval $[-1, 5]$.
   (b) Explain why the above method would fail if one wants to find the maximum/minimum value of f on the open interval $(-1, 5)$. Does f have a maximum or a minimum on $(-1, 5)$? (Hint: Use the graph of f to help you determine the answer.)
   (c) What if now one wants to find the maximum of f on $[-1, \infty)$? Does f have a maximum on $[-1, \infty)$?
   (d) What if now one wants to find the minimum of f on $[-1, \infty)$? Does f have a maximum on $[-1, \infty)$? (Hint: First show $f(x) \geq f(3)$ for all $x \in [3, \infty)$ (say using the first derivative of f). Then deduce $f(x) \geq f(-1)$ for all $x \in [-1, 3]$ (say from part (a)). Finally, conclude $f(x) \geq f(-1)$ for all $x \in [-1, \infty)$.)

4. Using the method in Question 2, find the absolute maximum and minimum of the following functions on the following closed and bounded intervals. You should find the points where the maximum / minimum are achieved, and the values of the function at these maximum / minimum points.
   (a) $f(x) = xe^{-x}$, on the interval $-1 \leq x \leq 2$.
   (b) $g(x) = (1 - x^2)^2$, on the interval $-2 \leq x \leq 2$.
   (c) $h(x) = \dfrac{1}{x} + \ln x$, on the interval $\dfrac{1}{2} \leq x \leq 4$
   (d) $f(x) = 2 - |x|$, on the interval $-1 \leq x \leq 4$
   (e) $g(x) = \begin{cases} -x^2 - 2x + 4, & \text{if } \; x \leq 1 \\ -x^2 + 6x - 4, & \text{if } \; x > 1 \end{cases}$, on the interval $[-2, 2]$
   (f) $h(x) = |x^3 - 9x|$, on the interval $[-1, 4]$.

Setting c' equal to zero gives

$$\begin{aligned}
500{,}000\,(20 - y) &= 300{,}000\sqrt{144 + (20 - y)^2}\\
\frac{5}{3}\left(20 - y\right) &= \sqrt{144 + (20 - y)^2}\\
\frac{25}{9}\left(20 - y\right)^2 &= 144 + (20 - y)^2\\
\frac{16}{9}\left(20 - y\right)^2 &= 144\\
(20 - y) &= \pm\frac{3}{4}\cdot 12 = \pm 9\\
y &= 20 \pm 9\\
y &= 11 \quad\text{or}\quad y = 29.
\end{aligned}$$

Only $y = 11$ lies in the interval of interest. The values of c at this one critical point an the endpoints are

$$\begin{aligned}
c(11) &= 10{,}800{,}000\\
c(0) &= 11{,}661{,}900\\
c(20) &= 12{,}000{,}000
\end{aligned}$$

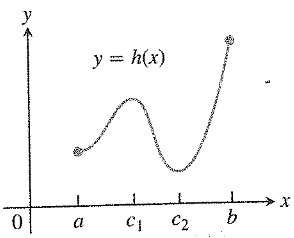
The least expensive connection costs \$10,800,000, and we achieve it by running the underwater to the point on shore 11 mi from the refinery.

EXERCISES 4.1

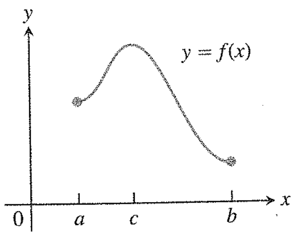
Finding Extrema from Graphs

In Exercises 1–6, determine from the graph whether the function has any absolute extreme values on $[a, b]$. Then explain how your answer is consistent with Theorem 1.

1.



2.



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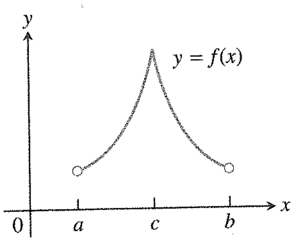

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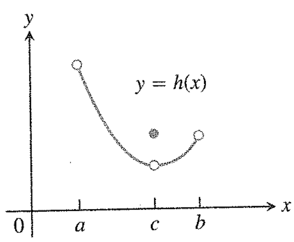

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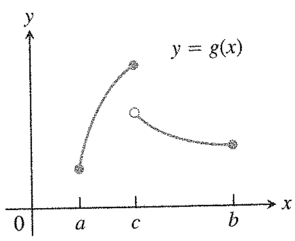

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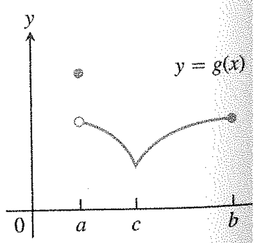
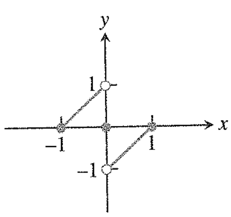


In Exercises 7–10, find the extreme values and where they occur

7.



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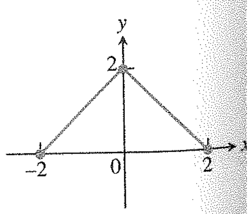
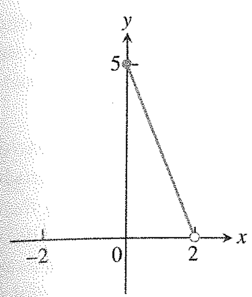
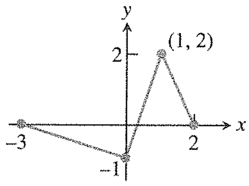


9.



10.



In Exercises 11–14, match the table with a graph.

11.

x	$f'(x)$
a	0
b	0
c	5

12.

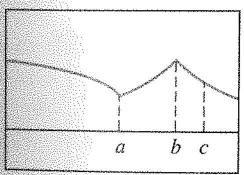
x	$f'(x)$
a	0
b	0
c	-5

13.

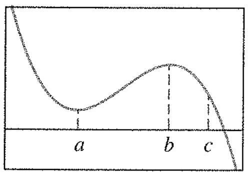
x	$f'(x)$
a	does not exist
b	0
c	-2

14.

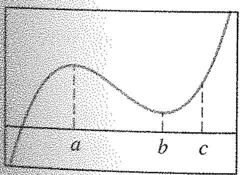
x	$f'(x)$
a	does not exist
b	does not exist
c	-1.7



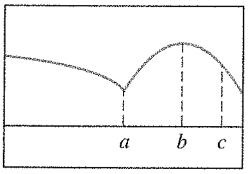
(a)



(b)



(c)



(d)

Absolute Extrema on Finite Closed Intervals

In Exercises 15–34, find the absolute maximum and minimum values of each function on the given interval. Then graph the function. Identify the points on the graph where the absolute extrema occur, and include their coordinates.

15. $f(x) = \frac{2}{3}x - 5, \quad -2 \le x \le 3$
16. $f(x) = -x - 4, \quad -4 \le x \le 1$
17. $f(x) = x^2 - 1, \quad -1 \le x \le 2$
18. $f(x) = 4 - x^2, \quad -3 \le x \le 1$
19. $F(x) = -\frac{1}{x^2}, \quad 0.5 \le x \le 2$
20. $F(x) = -\frac{1}{x}, \quad -2 \le x \le -1$
21. $h(x) = \sqrt[3]{x}, \quad -1 \le x \le 8$
22. $h(x) = -3x^{2/3}, \quad -1 \le x \le 1$
23. $g(x) = \sqrt{4 - x^2}, \quad -2 \le x \le 1$
24. $g(x) = -\sqrt{5 - x^2}, \quad -\sqrt{5} \le x \le 0$
25. $f(\theta) = \sin\theta, \quad -\frac{\pi}{2} \le \theta \le \frac{5\pi}{6}$
26. $f(\theta) = \tan\theta, \quad -\frac{\pi}{3} \le \theta \le \frac{\pi}{4}$
27. $g(x) = \csc x, \quad \frac{\pi}{3} \le x \le \frac{2\pi}{3}$
28. $g(x) = \sec x, \quad -\frac{\pi}{3} \le x \le \frac{\pi}{6}$
29. $f(t) = 2 - |t|, \quad -1 \le t \le 3$
30. $f(t) = |t - 5|, \quad 4 \le t \le 7$
31. $g(x) = xe^{-x}, \quad -1 \le x \le 1$
32. $h(x) = \ln(x + 1), \quad 0 \le x \le 3$
33. $f(x) = \frac{1}{x} + \ln x, \quad 0.5 \le x \le 4$
34. $g(x) = e^{-x^2}, \quad -2 \le x \le 1$

In Exercises 35–38, find the function's absolute maximum and minimum values and say where they are assumed.

35. $f(x) = x^{4/3}, \quad -1 \le x \le 8$
36. $f(x) = x^{5/3}, \quad -1 \le x \le 8$
37. $g(\theta) = \theta^{3/5}, \quad -32 \le \theta \le 1$
38. $h(\theta) = 3\theta^{2/3}, \quad -27 \le \theta \le 8$

Finding Extreme Values

In Exercises 39–54, find the extreme values of the function and where they occur.

39. $y = 2x^2 - 8x + 9$
40. $y = x^3 - 2x + 4$
41. $y = x^3 + x^2 - 8x + 5$
42. $y = x^3 - 3x^2 + 3x - 2$
43. $y = \sqrt{x^2 - 1}$
44. $y = \frac{1}{\sqrt{1 - x^2}}$
45. $y = \frac{1}{\sqrt[3]{1 - x^2}}$
46. $y = \sqrt{3 + 2x - x^2}$
47. $y = \frac{x}{x^2 + 1}$
48. $y = \frac{x + 1}{x^2 + 2x + 2}$
49. $y = e^x + e^{-x}$
50. $y = e^x - e^{-x}$
51. $y = x \ln x$
52. $y = x^2 \ln x$
53. $y = \cos^{-1}(x^2)$
54. $y = \sin^{-1}(e^x)$