

How is University mathematics different from high school?

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Impressions about high school mathematics

- ▶ Broadly divided into calculus, algebra and geometry
- ▶ Main emphasis: knowing how to calculate, and obtaining the 'correct answer'

How is University mathematics different from high school?

- ▶ Breadth
- ▶ Depth
- ▶ Focus

Breadth

- ▶ We still have calculus, algebra and geometry at the University.
- ▶ But they come in very different flavors:
 - ▶ Multivariable calculus, real analysis, complex analysis
 - ▶ study of symmetries, groups, rings, fields, commutative algebra
 - ▶ Algebraic geometry, Riemannian geometry, symplectic geometry, complex geometry, conformal geometry, contact geometry . . .
- ▶ On top of that, also many areas of mathematics that may be new to you:
 - ▶ functional analysis, harmonic analysis, convex analysis;
 - ▶ partial differential equations, mathematical physics;
 - ▶ numerical analysis, operations research, image processing, optimization, logistics, financial mathematics, game theory;
 - ▶ analytic number theory, algebraic number theory;
 - ▶ representation theory, supersymmetry;
 - ▶ stochastic processes, ergodic theory, dynamical systems;
 - ▶ differential topology, algebraic topology, low dimensional topology, knot theory . . .

Depth

- ▶ Not just about isolated theorems, but also about theories: e.g. Galois theory, measure theory, gauge theory, KAM theory . . .
- ▶ The deepest theorems often reveal connections between different areas of mathematics.
- ▶ Ultimately university mathematics will lead one to frontiers of current research (territories completely open and unknown)

Focus

- ▶ More emphasis on conceptual understandings than calculations (hence more abstract)
- ▶ More emphasis on definitions, theorems, proofs
- ▶ A higher demand for precise statements, logical deductions, and concise presentations
- ▶ More importantly, one learns to distinguish
 - ▶ **true** statements from **false** ones, and
 - ▶ **logical** reasonings from **illogical** ones
- ▶ More open-ended; hence creativity and imagination also play a much bigger role

What effect does this have about learning University mathematics?

- ▶ Memorization is very inefficient, and also less fun
- ▶ Needs to gain real understandings instead
- ▶ Needs to be critical about information you receive (even if that comes from a trusted source, or from yourself)
- ▶ Simple concrete (even extreme) examples often help, when it comes to understanding abstract definitions and theories
- ▶ Language skills become more important (in conveying an idea, or giving a clear presentation)
- ▶ Needs to learn through trial and error
(Guesswork plays a much more important role now. One needs to learn how to make good guesses. That means one should not be afraid of making a failed attempt, for it often take LOTS of attempts before one comes up with a good idea.)

Three stages of learning more advanced mathematics

- ▶ Initially one learns about calculations (1st-2nd year college)
- ▶ Gradually one shifts to the foundations of the subject, carefully establishing results taken for granted before (2nd-3rd year)
- ▶ Then one goes into more advanced topics, specializing in various areas of interest (3rd year onwards)

How can one learn mathematics well at a more advanced level?

- ▶ Learn to distinguish assumptions from conclusions
- ▶ Learn to test assumptions of theorems
(and see whether the theorems apply)
- ▶ Learn through examples and exercises
(*Math is not a spectator sport!* Hard work is rewarded)
- ▶ Learn from your friends (Math is very collaborative nowadays)
- ▶ Come up with examples and exercises of your own
- ▶ Discover your own proofs
- ▶ Be curious
- ▶ Ask questions! Even “dumb” ones. And answer them.
- ▶ Don't be afraid of challenging conventional wisdom
- ▶ Enjoy mathematics!