MATH4060 Exercise 2

Due Date: October 2, 2018.

The questions are from Stein and Shakarchi, *Complex Analysis*, unless otherwise stated.

Chapter 1. Exercise 4, 5, 26.

Chapter 2. Exercise 4, 5.

Chapter 3. Exercise 15(a)(d), 22.

Chapter 4. Exercise 1, 2, 3, 6, 7.

Additional Exercise. Show that if $0 < a < 1$, then

$$\int_0^\infty \frac{v^{a-1}}{1+v} dv = \frac{\pi}{\sin \pi a}.$$

(Hint: Consider, for a small $\delta > 0$, the contour $\gamma_1 + \gamma_2 - \gamma_3 - \gamma_4$, where

$$\begin{aligned}
\gamma_1 &= \{t + i\delta \colon \varepsilon \le t \le R\}, \\
\gamma_2 &= \{(R^2 + \delta^2)^{1/2} e^{i\theta} \colon \arctan(\delta/R) \le \theta \le 2\pi - \arctan(\delta/R)\}, \\
\gamma_3 &= \{t - i\delta \colon \varepsilon \le t \le R\}, \\
\gamma_4 &= \{(\varepsilon^2 + \delta^2)^{1/2} e^{i\theta} \colon \arctan(\delta/\varepsilon) \le \theta \le 2\pi - \arctan(\delta/\varepsilon)\},
\end{aligned}$$

and consider a suitable branch of $v \mapsto v^{a-1}$ for v away from the non-negative real axis.)