THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MMAT5230 Mathematics for Logistics (Fall 2015)
Homework 4
Due Date: 9th December, 2015

Name: ______________________________ Student No.: ____________________

Class: ______________________________ Final Result: ____________________

I acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained in the website http://www.cuhk.edu.hk/policy/academichonesty/

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Signature Date

General Directions: You must show all work and document any assumptions to receive full credit. Make sure to clearly define your decision variables and label your constraints. All problems are to be done by hand unless otherwise stated.

1. Consider the integer linear programming problem:

$$\text{Maximise } z = 2x_1 + 2x_2$$

subject to

$$\begin{array}{rcrcl} 5x_1 & + & 3x_2 & \leq & 8 \\ x_1 & + & 2x_2 & \geq & 4 \end{array}$$

$$x_1, x_2 \geq 0 \text{ and } x_1, x_2 \ \text{ are non} - \text{negative integers.}$$

The optimal tableau is

			$\mathbf{y}_1$	$\mathbf{y}_2$	$\mathbf{y}_3$	$\mathbf{y}_4$
C_B	$\mathbf{X}_B$		x_1	x_2	s_1	s_2
2	x_1	4/3	7/3	0	2/3	1
2	x_2	8/3	5/3	1	1/3	0
$z_j - c_j$	$z(\mathbf{X}_B) = 16/3$		4/3	0	2/3	0

Answer the following question:

(a) Solve the above integer programming problem using Gomory's cutting plane method.

(b) Also verify graphically that each of the Gomory cut constraint is really deleting a part of the feasible region but not deleting any of the integer feasible points.

2. Show that the following integer programming problem using Gomory's cutting plane method has the optimal integer-valued solution as $(x_1, x_2) = (0, 2)$ with a maximum value of $z = 2$:

$$\text{Maximise } z = x_1 + x_2$$

subject to

$$\begin{array}{rcrcl} 3x_1 & + & 2x_2 & \leq & 5 \\ & & x_2 & \leq & 2 \end{array}$$

$$x_1, x_2 \geq 0 \text{ and } x_1, x_2 \text{ are integers.}$$

3. Solve the following mixed integer programming problems using Gomory's cutting plane method:

$$\text{Maximise } z = 4x_1 + 6x_2 + 2x_3$$

subject to

$$\begin{array}{rcrcrcl} 4x_1 & - & 4x_2 & & & \leq & 5 \\ -x_1 & + & 6x_2 & & & \leq & 5 \\ -x_1 & + & x_2 & + & x_3 & \leq & 5 \end{array}$$

$$x_1, x_2, x_3 \geq 0 \text{ and } x_1, x_3 \text{ are integers.}$$

The optimal tableau is

			$\mathbf{y}_1$	$\mathbf{y}_2$	$\mathbf{y}_3$	$\mathbf{y}_4$	$\mathbf{y}_5$	$\mathbf{y}_6$
C_B	$\mathbf{X}_B$		x_1	x_2	x_3	s_1	s_2	s_3
4	x_1	5/2	1	0	0	3/10	1/5	0
6	x_2	5/4	0	1	0	1/20	1/5	0
2	x_3	25/4	0	0	1	1/4	0	1
$z_j - c_j$	$z(\mathbf{X}_B) = 30$		0	0	0	2	2	2

4. Use Branch and Bound technique to solve the following LPPs:

$$\text{Mimimise } z = 3x_1 + 2.5x_2$$

subject to

$$\begin{array}{rcrcl} x_1 & + & 2x_2 & \geq & 20 \\ 3x_1 & + & 2x_2 & \geq & 50 \end{array}$$

$$x_1, x_2 \geq 0 \text{ and } x_1, x_2 \text{ are non-negative integers.}$$

5. (Optional) Use Gomory's cutting plane method and Branch and Bound technique to solve the following LPP and compare the solutions:

$$\text{Maximise } z = x_1 + 2x_2 + x_3$$

subject to

$$2x_1 + 3x_2 + 3x_3 \le 11$$

$$x_1, x_2, x_3 \ge 0 \text{ and } x_1, x_2, x_3 \text{ are non} - \text{negative integers.}$$

6. (Optional) Consider the following standard-form LPP:

$$\text{Minimise } z = 14x_1 + 3x_2 + 5x_3$$

subject to

$$\begin{array}{rcrcrcl} 2x_1 & & & + & 5x_3 & = & 1 \\ x_1 & + & x_2 & & & = & 1 \end{array}$$

$$x_1, x_2, x_3 \ge 0.$$

(a) Determine the direction of most rapid objective function improvement for any solution z.

(b) Compute the projection matrix P for the main equality constraints.

(c) Apply your P to project the direction of part (a).

(d) Verify that the result of part (c) is improving and feasible at any interior point solution.

(e) Describe the sense in which the direction of part (c) is good for improving search at any interior point solution.

7. (Optional) Develop a model for minimizing the number of persons to be sent to a conference from amongst a group of 12 students and under the following constraints:

(a) At least one student from each department must attend the conference.

(b) Parity must exist between graduate and undergraduate students.

(c) For ease of formulation, a number is assigned to each student.

The data are as in the following table:

Category	ID Number
Graduate	1, 3, 6, 7, 9, 12
Undergraduate	2, 4, 5, 8, 10, 11
Maths Dept	1, 2, 3, 12
CS Dept	4, 5, 9
Phys Dept	7, 8
Eng Dept	6, 10, 11

8. (Optional) A university is considering installing Wi-Fi facilities for the students. There are seven possible locations for the facilities. For this purpose, the campus has been divided into eight areas as highlighted in the following table:

Areas	Could be served by Locations
Academic Building	1, 3, 4, 5
Art Museum	2, 4, 6, 7
University Health Centre	1, 2, 5, 6, 7
University Library	1, 4, 7
University Sports Centre	1, 3, 4, 6
Engineering Building	1, 3, 5, 7
Car Park	2, 3, 4, 6
Staff Club	1, 3, 5 ,6

Write a model that will minimise the number of Wi-Fi locations such that each area is covered by a least three facilities.