THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MMAT5230 Mathematics for Logistics (Fall 2015)
Homework 3
Due Date: 25th November, 2015

Name: _____ Student No.: _____

Class: _____ Final Result: _____

I acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained in the website http://www.cuhk.edu.hk/policy/academichonesty/	
_____	_____
Signature	Date

General Directions: You must show all work and document any assumptions to receive full credit. Make sure to clearly define your decision variables and label your constraints. All problems are to be done by hand unless otherwise stated.

1. Apply the principle of duality that consists of two steps:

Step 1: Construct the dual

Step 2: Use the simplex method to check the principle of duality

to solve the LPPs:

(a)

$$\begin{aligned} \text{Minimise} \quad & z = 2x_1 + x_2 \\ \text{subject to} \quad & \begin{cases} 3x_1 + x_2 \geq 3 \\ 4x_1 + 3x_2 \geq 6 \\ x_1 + 2x_2 \geq 3 \end{cases} \\ \text{and} \quad & x_1, x_2 \geq 0. \end{aligned}$$

(b)

$$\begin{aligned} \text{Minimise} \quad & z = (15/2)x_1 - 3x_2 \\ \text{subject to} \quad & \begin{cases} 3x_1 - x_2 - x_3 \geq 3 \\ x_1 - x_2 + x_3 \geq 2 \end{cases} \\ \text{and} \quad & x_1, x_2, x_3 \geq 0. \end{aligned}$$

2. Solve the following LPP by simplex method

$$(\mathsf{P}): \quad \begin{cases} \text{Minimize} & z = x_1 + 2x_2 + 3x_3 \\ \text{subject to} & \begin{cases} x_1 + 2x_2 + 3x_3 \geq 4 \\ x_1 + x_2 + 2x_3 \geq 8 \\ x_2 - x_3 \geq 2 \end{cases} \\ \text{and} & x_1, x_2, x_3 \geq 0. \end{cases}$$

Also write the dual of the problem (P) and solve. Read the solution of each problem from the final simplex table of the other.

3. Solve the following problem:

$$(\mathsf{P}): \quad \begin{cases} \text{Maximise} & z = x_1 + x_2 + 3x_3 \\ \text{subject to} & \begin{cases} 3x_1 + 2x_2 + x_3 \leq 3 \\ 2x_1 + x_2 + 2x_3 \leq 2 \end{cases} \\ \text{and} & x_1, x_2, x_3 \geq 0 \end{cases}$$

using the following strategy:

(a) Write down the dual problem (D).

(b) Solve the dual problem using the graphical method.

(c) Using complementary slackness, derive an optimal solution for (P).

4. Solve the following linear programming problems using dual simplex method.

(a)

$$\begin{cases} \text{Maximise} & z = -3x_1 - 2x_2 \\ \text{subject to} & \begin{cases} x_1 + x_2 \geq 1 \\ x_1 + x_2 \leq 7 \\ x_1 + 2x_2 \geq 10 \\ x_2 \leq 3 \end{cases} \\ \text{and} & x_1, x_2 \geq 0. \end{cases}$$

(b)

$$\begin{cases} \text{Minimize} & z = 3x_1 + 2x_2 + x_3 + 4x_4 \\ \text{subject to} & \begin{cases} 2x_1 + 4x_2 + 5x_3 + x_4 \geq 10 \\ 3x_1 - x_2 + 7x_3 - 2x_4 \geq 2 \\ 5x_1 + 2x_2 + x_3 + 6x_4 \geq 15 \end{cases} \\ \text{and} & x_1, x_2, x_3, x_4 \geq 0. \end{cases}$$

5. (Optional) A firm uses three machines to manufacture three products.

   - Each unit of product A requires three hours on machine I, two hours on machine II, and one hour on machine III.
   - Each unit of product B requires four hours on machine I, one hour on machine II, and three hours on machine III.
   - Each unit of product C requires two hours on each of the three machines.

   The contribution margin of the three products is \$ 30, \$ 40, and \$ 35 per unit, respectively. The machine hours available on the three machines are 90, 54, and 93, respectively.

   (a) Formulate the above as a LPP and solve for the maximum product, using the simplex method.

   (b) Write the dual to the LPP.

   (c) Obtain the optimum value of the dual variables and verify the primal and the dual problems have the same objective function values.

6. (Optional) A firm produces three products A, B, and C. Unit contributions of the products are \$5, 10, and 8, respectively.

   - Each unit of product A requires 3 kg of material, 5 machine hours and 2 labour hours;
   - each unit of product B requires 4 kg of material, 4 machine hours and 4 labour hours;
   - each unit of product C requires 2 kg of material, 4 machine hours and 5 labour hours.

   Everyday 60 kg of material, 72 machine hours and 100 labour hours are available. Find the best production strategy, investigate the effect on the solution of the following:

   (a) An increase of 12 machine hours;

   (b) A decrease of 6 kg of material;

   (c) Three units of product A are to be produced.