THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MMAT5230 Mathematics for Logistics (Fall 2015)
Homework 2
Due Date: 28th October, 2015

Name: _____ Student No.: _____

Class: _____ Final Result: _____

I acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained in the website http://www.cuhk.edu.hk/policy/academichonesty/	
_____	_____
Signature	Date

General Directions: You must show all work and document any assumptions to receive full credit. Make sure to clearly define your decision variables and label your constraints. All problems are to be done by hand unless otherwise stated.

Please attempt to solve all the problems. Your solutions of problems 1, 3, 5, 6, 7, 8 are to be submitted.

Problems 2, 4, 9 are strongly recommended for you to study, though you are not required to submit their solutions.

1. Use the Big M method to solve the LPP:

$$\text{Maximize } z = x_1 + 2x_2 + 3x_3$$

subject to

$$\begin{array}{rcrcrcl} x_1 & - & x_2 & + & x_3 & \geq & 4 \\ x_1 & + & x_2 & + & 2x_3 & \leq & 8 \\ x_1 & & & - & x_3 & \geq & 2 \end{array}$$

$$x_1, x_2, x_3 \geq 0.$$

Determine the optimal basic feasible solution to the given above LPP $(x_1^*, x_2^*, x_3^*) = (18/5, 6/5, 8/5)$ with the maximize value z, i.e., $z^* = 54/5$ or not.

2. (Optional) Use the Big M method to solve the LPP:

$$\text{Minimize } z = x_1 + x_2 + x_3$$

subject to

$$\begin{array}{rcrcrcl} x_1 & - & 3x_2 & + & 4x_3 & = & 5 \\ x_1 & - & 2x_2 & & & \leq & 3 \\ & & 2x_2 & - & x_3 & \geq & 4 \end{array}$$

$$x_1, x_2, \geq 0 \text{ and } x_3 \text{ is unrestricted.}$$

3. Solve the following system of linear equations by using the simplex method.

$$\begin{array}{rcrcrcrcr} x_1 & + & x_2 & + & 4x_3 & + & 2x_4 & = & 9 \\ x_1 & + & x_2 & + & 2x_3 & - & x_4 & = & 3 \\ 2x_1 & + & 2x_2 & + & 4x_3 & + & x_4 & = & 12 \end{array}$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

4. (Optional) Use the two-phase simplex method to solve the LPP:

$$\text{Maxmize } z = -x_1 - x_2$$

subject to

$$\begin{array}{rcrcr} 3x_1 & + & 2x_2 & \geq & 30 \\ -2x_1 & + & 3x_2 & \leq & -30 \\ x_1 & + & x_2 & \leq & 5 \end{array}$$

$$x_1, x_2 \geq 0.$$

5. Use the two-phase simplex method to solve the LPP:

$$\text{Minimize } z = 2x_1 + x_2 + 7x_3 + 5x_4$$

subject to

$$\begin{array}{rcrcrcrcr} - & x_1 & + & x_2 & + & 3x_3 & + & 2x_4 & \geq & 135 \\ & x_1 & - & 2x_2 & + & x_3 & + & 2x_4 & \geq & 105 \\ & & & x_2 & & & + & x_4 & \leq & 10 \end{array}$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

6. Use the two-phase simplex method to solve the LPP:

$$\text{Maximise } z = 5x_1 - 4x_2 + 3x_3$$

subject to

$$\begin{array}{rcrcrcl} 2x_1 & + & x_2 & - & 6x_3 & = & 20 \\ 6x_1 & + & 5x_2 & + & 10x_3 & \leq & 76 \\ 8x_1 & - & 3x_2 & + & 6x_3 & \leq & 50 \end{array}$$

$$x_1, x_2, x_3 \geq 0.$$

Determine the optimal basic feasible solution to the given above LPP $(x_1^*, x_2^*, x_3^*) = (55/7, 30/7, 0)$ with the maximize value z, i.e., $z^* = 155/7$ or not.

7. Use the revised simplex method for standard form I or for standard form II to solve the LPP:

(a)

$$\text{Maximise } z = 3x_1 + 5x_2$$

subject to

$$\begin{aligned} x_1 &\leq 4 \\ x_2 &\leq 6 \\ 3x_1 + 2x_2 &\leq 18 \end{aligned}$$

$$x_1, x_2 \geq 0.$$

(b)

$$\text{Minimize } z = x_1 + 2x_2$$

subject to

$$\begin{aligned} 2x_1 + 5x_2 &\geq 6 \\ x_1 + x_2 &\geq 2 \end{aligned}$$

$$x_1, x_2 \geq 0.$$

8. Show that there does not exist any feasible solution to the following LPP:

$$\text{Minimize } z = x_1 - 2x_2 - 3x_3$$

subject to

$$\begin{aligned} -2x_1 + x_2 + 3x_3 &= 2 \\ 2x_1 + 3x_2 + 4x_3 &= 1 \end{aligned}$$

$$x_1, x_2, x_3 \geq 0.$$

9. (Optional)

(a) Let $x_1 = 1,\ x_2 = 0,\ x_3 = 1$ be a feasible solution of the LPP:

$$\text{Minimize } z = 2x_1 + 3x_2 + 4x_3$$

subject to

$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ x_1 - x_2 + x_3 &= 0 \end{aligned}$$

$$x_1, x_2, x_3 \geq 0.$$

Show that the given feasible solution is not basic.

(b) Use the revised simplex method to solve the LPP:

$$\text{Maximise } z = 6x_1 + 3x_2 + 4x_3 - 2x_4 + x_5$$

subject to

$$\begin{aligned} 2x_1 + 3x_2 + 3x_3 + x_4 &= 10 \\ x_1 + 2x_2 + 3x_3 + x_5 &= 8 \end{aligned}$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0.$$

(c) Solve the following LPP:

$$\text{Maximise } z = 3x_1 + 5x_2 + 4x_3$$

subject to

$$\begin{array}{rcrcrcr} 2x_1 & + & 3x_2 & & & \leq & 8 \\ & & 2x_2 & + & 5x_3 & \leq & 10 \\ 3x_1 & + & 2x_2 & + & 4x_3 & \leq & 15 \end{array}$$

$$x_1, x_2, x_3 \geq 0.$$

using

i. pivoting in the simplex method and

ii. the LU factorization.