THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MMAT5230 Mathematics for Logistics (Fall 2015)
Homework 1
Due Date: 30^{th} September, 2015

General Directions: You must show all work and document any assumptions to receive full credit. Make sure to clearly define your decision variables and label your constraints. All problems are to be done by hand unless otherwise stated.

1. (a) Find all the basic solutions of the equations:

$$\begin{array}{rcrcrcrcr} 2x_1 & + & 3x_2 & + & x_3 & & & = & 12 \\ 2x_1 & + & x_2 & & & + & x_4 & = & 8. \end{array}$$

   Also find which of the basic solutions are:

   i. basic feasible, and
   ii. non-degenerate basic feasible.

   (b) Use 1(a) to solve the following LPP:

$$\text{Maximise } z = 6x_1 + 7x_2$$

   subject to

$$\begin{array}{rcrcl} 2x_1 & + & 3x_2 & \leq & 12 \\ 2x_1 & + & x_2 & \leq & 8 \\ & & x_1, x_2 & \geq & 0. \end{array}$$

2. A retired person wants to invest up to an amount \$30,000 in fixed income securities. Her broker recommends investing in two bounds: Bond A yielding 7 per cent and Bond B yielding 10 per cent. After some consideration, she decides to invest at most \$ 12,000 in Bond B and at least \$6000 in Bond A. She also wants the amount invested in Bond A to be at least equal to the amount invested in Bond B. What should the broker recommend if the investor wants to maximize her turn on investment? Hint:

   (a) Formulate this problem as a linear programming problem.
   (b) Find the optimal solution using the graphical method.

3. Answer the following questions:

   (a) Solve the following by the graphical method

$$\text{Minimize } z = 6x_1 + 4x_2$$

   subject to

$$\begin{aligned} 7x_1 + 5x_2 &\le 35 \\ 5x_1 + 7x_2 &\le 35 \\ 4x_1 + 3x_2 &\ge 12 \\ 3x_1 + x_2 &\ge 3 \\ x_1, x_2 &\ge 0. \end{aligned}$$

   (b) Solve the following by the graphical method

$$\text{Maximise } z = 6x_1 + 2.5x_2$$

   subject to

$$\begin{aligned} 7x_1 + 9x_2 &\le 63 \\ 12x_1 + 5x_2 &\le 60 \\ x_1, x_2 &\ge 0. \end{aligned}$$

   (c) Solve the following by the graphical method

$$\text{Maximise } z = 3x_1 + 4x_2$$

   subject to

$$\begin{aligned} -3x_1 + 2x_2 &\le 6 \\ -4x_1 + 3x_2 &\le 18 \\ x_1, x_2 &\ge 0. \end{aligned}$$

4. Use simplex method to solve the LPP

$$\text{Maximise } z = 3x_1 + 5x_2$$

subject to

$$\begin{aligned} x_1 &\le 4 \\ x_2 &\le 6 \\ 3x_1 + 2x_2 &\le 18 \\ x_1, x_2 &\ge 0. \end{aligned}$$

5. Use simplex method to solve the LPP

$$\text{Minimize } z = x_1 - x_2 + 3x_3$$

subject to

$$\begin{aligned} x_1 + x_2 + x_3 &\le 10 \\ 2x_1 - x_3 &\le 2 \\ 2x_1 - 2x_2 + 3x_3 &\le 0 \end{aligned}$$

$$x_1, x_2, x_3 \ge 0.$$

6. Determine if the following LPP

$$\text{Maximise } z = 15x_1 + 6x_2 + 3x_3 + 2x_4$$

subject to

$$\begin{array}{lcccccccl} 2x_1 & + & x_2 & + & 5x_3 & + & 0.6x_4 & \leq & 10 \\ 3x_1 & + & x_2 & + & 3x_3 & + & 0.25x_4 & \leq & 12 \\ 7x_1 & & & & & + & x_4 & \leq & 35 \end{array}$$

$$x_1, x_2, x_3, x_4 \geq 0$$

has an unbounded solution or not.

7. Let $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_k, \in \mathbb{R}^n$. The linear combination

$$\lambda_1 \mathbf{x}_1 + \lambda_2 \mathbf{x}_2 + \cdots + \lambda_k \mathbf{x}_k,$$

$\lambda_1, \lambda_2, \cdots, \lambda_k$ are real numbers, is said to be a convex combination of $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_k$, if $\lambda_i \geq 0$ for all i and

$$\sum_{i=1}^{k} \lambda_i = 1.$$

Show that the set of all convex combinations of a finite number of n-vectors

$$\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_k,$$

is a convex set.

8. Recall that a half-space in $\mathbb{R}^n$ is a subset of the form $\{\mathbf{x} | \ \mathbf{a}^T \mathbf{x} \leq b\}$ or $\{\mathbf{x} | \ \mathbf{a}^T \mathbf{x} \geq b\}$. A convex polyhedron is the (possibly empty) intersection of a given collection of half-spaces.

   (a) Show that a convex polyhedron is indeed convex.

   (b) Show that a feasible region of an LP problem is a convex polyhedron.