Assignment 7.

1. $$f = \frac{1}{2}\lg E = \frac{1}{2}\lg\frac{16}{[4-(x^2+y^2)]^2} = -\lg\frac{4-(x^2+y^2)}{4}$$

$$\therefore\ f_x = -\frac{4\cdot(-\frac{1}{2}x)}{4-(x^2+y^2)} = \frac{2x}{4-(x^2+y^2)}$$

$$f_{xx} = \frac{4-(x^2+y^2)+2x^2}{[4-(x^2+y^2)]^2}\cdot 2 = \frac{4+x^2-y^2}{[4-(x^2+y^2)]^2}\cdot 2$$

$\therefore$ Similarly, $$f_{yy} = \frac{4-x^2+y^2}{[4-(x^2+y^2)]^2}\cdot 2$$

$$\therefore\ K = -\frac{1}{E}\Delta f = -\frac{[4-(x^2+y^2)]^2}{16}\cdot\frac{16}{[4-(x^2+y^2)]^2} = -1$$

2. let $\alpha(t) = (r\cos(u), r\sin(u), v(t))$ be a geodesic and p.b.a.l.

$\therefore 1 = |\alpha'(t)|^2 = r^2(u')^2 + (v')^2$

$\because X(u,v) = (r\cos u, r\sin u, v)$

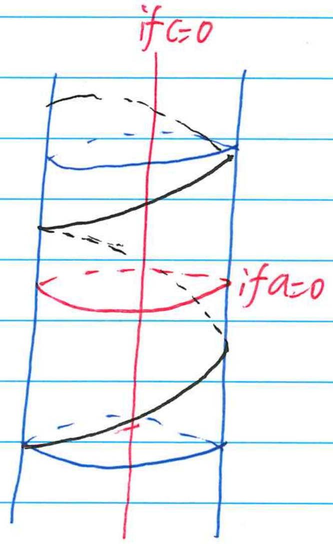
$\therefore X_u = (-r\sin u, r\cos u, 0)$, $\quad X_v = (0,0,1)$

$$(g_{ij}) = \begin{bmatrix} r^2 & 0 \\ 0 & 1 \end{bmatrix}$$

$\because r$ is a constant

$\therefore \Gamma_{ij}^k \equiv 0$ for $\forall i,j,k$

$$\therefore \begin{cases} u''(t)=0 \\ v''(t)=0 \end{cases} \Rightarrow \begin{cases} u(t) = at+b \\ v(t) = ct+d \end{cases}$$



$\because 1 = r^2(u')^2 + (v')^2$

$\therefore 1 = r^2a^2 + c^2$

$\therefore \alpha(t) = \langle r\cos(at+b), r\sin(ct+d), ct+d\rangle$ where $1 = r^2a^2+c^2$

3. $\because$ α is a pregeodesic $\quad \tau = F(t) = \int \exp(\int \lambda) \Rightarrow \frac{d\tau}{dt} = F' = e^{f(t)}$

$\therefore$ $$\begin{cases} \frac{d^2u^1}{dt^2} + \Gamma^1_{ij}\frac{du^i}{dt}\cdot\frac{du^j}{dt} = \lambda\frac{du^1}{dt} \\ \frac{d^2u^2}{dt^2} + \Gamma^2_{ij}\frac{du^i}{dt}\cdot\frac{du^j}{dt} = \lambda\frac{du^2}{dt} \end{cases}$$

$\therefore$ $$\frac{du^1}{d\tau} = \frac{du^1}{dt}\cdot\frac{\partial t}{\partial \tau} = \frac{du^1}{dt}\cdot\frac{dt}{d\tau} = \frac{du^1}{dt}\cdot e^{-f(t)}$$

$$\frac{d^2u^1}{d\tau^2} = \frac{d}{d\tau}\left(\frac{du^1}{dt}\right)\cdot e^{-f(t)} + \frac{du^1}{dt}\cdot e^{-f(t)}(-f'(t))\cdot\frac{dt}{d\tau}$$

$$= \frac{d^2u^1}{dt^2}\cdot e^{-2f} - e^{-2f}\frac{du^1}{dt}\cdot\lambda$$

similarly, $$\frac{d^2u^2}{d\tau^2} = \frac{d^2u^2}{dt^2}e^{-2f} - e^{-2f}\lambda\frac{du^2}{dt}$$

$\therefore$ $$\frac{d^2u^1}{d\tau^2} + \Gamma^1_{ij}\frac{du^i}{d\tau}\cdot\frac{du^j}{d\tau} = e^{-2f}\left(\frac{d^2u^1}{dt^2} + \Gamma^1_{ij}\frac{du^i}{dt}\frac{du^j}{dt} - \lambda\frac{du^1}{dt}\right)$$

$$= e^{-2f}\left(\lambda\frac{du^1}{dt} - \lambda\frac{du^1}{dt}\right) = 0$$

similarly, $$\frac{d^2u^2}{d\tau^2} + \Gamma^2_{ij}\frac{du^i}{d\tau}\cdot\frac{du^j}{d\tau} = 0$$

$\therefore$ $\alpha(t(\tau))$ is a geodesic.