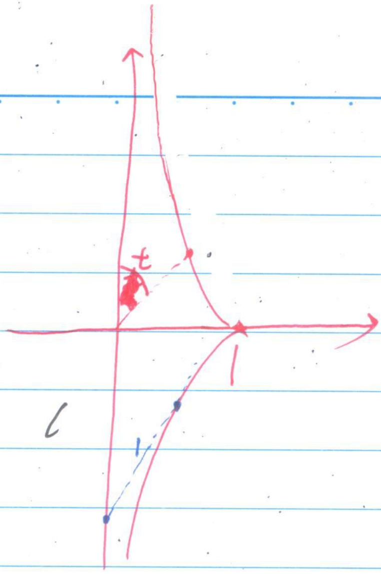
Assignment1:

(1) $\alpha(t) = (\sin t, \cos t + \log \tan\frac{t}{2})$

$\Rightarrow \alpha'(t) = (\cos t, -\sin t + \frac{1}{\sin t})$

(a) $\alpha' = 0$ for $t \in (0, \pi)$

$\Leftrightarrow t = \frac{\pi}{2}$



(b) denote the tangent line of α at $\alpha(t)$ be l

$\because \alpha' = (\cos t, -\sin t + \frac{1}{\sin t})$

$\therefore$ the intersection of l and y-axis is

$(0, \log \tan\frac{t}{2})$ for $t \neq \frac{\pi}{2}$

$\therefore$ the length of the segment of the tangent of α between the point of tangency and the y-axis is

$\sqrt{\sin^2 t + \cos^2 t} = 1$ for $t \neq \frac{\pi}{2}$

• Remark: • if the curve is C^1 at $\alpha(\frac{\pi}{2})$

then the tangent lines of this curve is continuous w.r.t t

$\therefore$ For $t = \frac{\pi}{2}$, the answer is true

• if the curve is only C^0 at $\alpha(\frac{\pi}{2})$

then there is no tangent line at $\alpha(\frac{\pi}{2})$

(2) "$\Rightarrow$" $\because \langle T, u\rangle = \cos\theta_0$ is a constant

$\therefore \langle T', u\rangle = 0$ since u is a constant

$k\langle N, u\rangle = 0$

$\therefore \langle N, u\rangle = 0$ since $k \neq 0$

$\therefore u = |u|(\cos\theta_1 T + \sin\theta_1 B)$ for some θ

$\because \langle T, u\rangle = \cos\theta_0$ $\therefore \theta$ is a constant

differentiate

$0 = \cos\theta_1 T' + \sin\theta_1 B'$

$= \cos\theta_1 kN - \sin\theta_1 \tau N$

$\therefore \frac{k}{\tau}$ is a constant

"$\Leftarrow$" $\because \frac{k}{\tau}$ is constant

$\therefore \frac{k}{\tau} = \frac{\sin\theta_2}{\cos\theta_2}$ for some θ_2

$\therefore (\cos\theta_2 T + \sin\theta_2 B)' = 0$

$\therefore$ Let $\vec{u} = \cos\theta_2 T + \sin\theta_2 B$

then $\langle T, \vec{u}\rangle$ is constant.

(3) "$\Rightarrow$" we can assume the center is the origin

$\therefore |\alpha|$ is a constant

$\therefore (|\alpha|^2)''' = 0$

$\therefore \alpha = -\rho N - \rho' \lambda \beta$ Or $|\alpha|^2 = \rho^2 + (\rho')^2 \lambda^2$

differentiate again, we will get the desire equation

[T, N, B are linearly independent
  $aT + bN + cB = 0 \Rightarrow a = b = c = 0$] which will give us $[\rho^2 + (\rho')^2\lambda^2]' \equiv 0$

"$\Leftarrow$" Let $\beta = \alpha + \rho N - \rho' \lambda B$

then $\beta' = 0$ by assumption $\therefore \beta$ is a fixed point

then $\frac{d}{ds} |\alpha - \beta|^2 = 0$

$\therefore |\alpha - \beta| = R$ for some constant R

$\therefore \alpha(s)$ lies on a sphere.