Assignment 6, Due Friday Nov 2, 2018

(1) Prove that if $\mathbf{X}$ is an orthogonal parametrization, i.e. $F = 0$, then the Gaussian curvature is given by:

$$K = -\frac{1}{2\sqrt{EG}}\left[\left(\frac{E_v}{\sqrt{EG}}\right)_v + \left(\frac{G_u}{\sqrt{EG}}\right)_u\right].$$

Suppose in addition $E = G$ everywhere, then

$$K = -e^{-2f}\Delta f$$

where f is such that $E = e^{2f}$ (i.e. $f = \frac{1}{2}\log E$), and Δ is the Laplacian operator:

$$\Delta = \frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2}.$$

(2) Compute the Christoffel symbols for a surface of revolution:

$$\mathbf{X}(u^1, u^2) = (f(u^2)\cos u^1, f(u^2)\sin u^1, g(u^2))$$

with $f > 0$.

(3) Verify that the surfaces:

$$\mathbf{X}(u, v) = (u\cos v, u\sin v, \log u)$$

and

$$\mathbf{Y}(u, v) = (u\cos v, u\sin v, v)$$

have equal Gaussian curvature at that points $\mathbf{X}(u, v), \mathbf{Y}(u, v)$ but the coefficients of the first fundamental forms at points $\mathbf{X}(u, v), \mathbf{Y}(u, v)$ are not the same.