Assignment 3, Due 27/9/2018

(1) Prove that the definition of tangent space is independent of the choice of parametrization.

(2) Consider the stereographic projection by $\mathbf{X} : \mathbb{R}^2 \to \mathbb{R}^3$ given by

$$\mathbf{X}(u, v) = \left(\frac{2u}{1 + u^2 + v^2}, \frac{2v}{1 + u^2 + v^2}, \frac{-1 + u^2 + v^2}{1 + u^2 + v^2} \right).$$

Show that this is a regular surface patch with image being the unit sphere $\mathbb{S}^2$ with the north pole $(0, 0, 1)$ deleted. Find also the coefficients of the first fundamental form.

(3) Consider the sphere parametrized by spherical coordinates:

$$\mathbf{X}(u, v) = (\sin v \cos u, \sin v \sin u, \cos v)$$

with $-\pi < u < \pi, 0 < v < \pi$. Find the length of the curve α given by $u = u_0$ and $a \le v \le b$ with $0 < a < b < \pi$. (That is $\alpha(t) = (\sin t \cos u_0, \sin t \sin u_0, \cos t)$, with $a \le t \le b$.) Let $\beta(t)$ be another curve joining $\alpha(a)$ to $\alpha(b)$ on the surface, i.e. $\beta(t) = \mathbf{X}(u(t), v(t))$, $a \le t \le b$ with $\beta(a) = \alpha(a)$, $\beta(b) = \alpha(b)$. Show that $\ell(\beta) \ge \ell(\alpha)$.