Assignment 2, Due Friday, 21/9/2018

(1) Show that part of the hyperboloid of one sheet:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

can be realized as a regular surface surface patch with following is a parametrization:

 $\mathbf{X}(u, v) = (a \cosh u \cos v, b \cosh u \sin v, c \sinh u).$

Find the largest domain on (u, v) plane such that **X** is one to one.

(2) Let \mathbf{S}^1 be the unit circle $x^2 + y^2 = 1$. Let $\alpha(s), 0 \le s \le 2\pi$, be a parametrization of \mathbf{S}^1 by arc length. Let $\mathbf{w}(s) = \alpha'(s) + e_3$ where $e_3 = (0, 0, 1)$. Show the ruled surface

$$\mathbf{X}(s,v) = \alpha(s) + v\mathbf{w}(s)$$

with $-\infty < v < \infty$, is part of the hyperboloid $x^2 + y^2 - z^2 = 1$. Is **X** a surjective map to the hyperboloid? Is **X** injective? Does **X** has rank 2 for $0 < s < 2\pi$, $v \in \mathbb{R}$?

- (3) Find a parametrization for the catenoid, which is obtained by revolving the catenary $y = \cosh x$ about the x-axis.
- (4) The Enneper's surface is defined by

$$\mathbf{X}(u,v) = (u - \frac{u^3}{3} + uv^2, v - \frac{v^3}{3} + vu^2, u^2 - v^2).$$

Show that this a regular surface patch for $u^2 + v^2 < 3$. Also find two points on the circle $u^2 + v^2 = 3$ such that they have the same image under **X**.