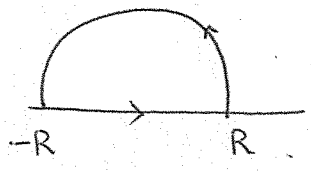
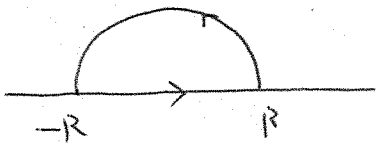
Lecture 20 and 21

Improper Integration.

Type I. $\int_{-\infty}^{+\infty} \frac{P(x)}{Q(x)}\,dx$.

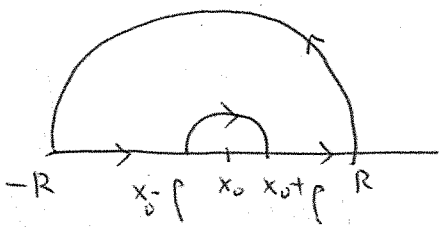


Type II. $\int_{-\infty}^{+\infty} \frac{P(x)}{Q(x)} \sin ax\,dx$, $\int_{-\infty}^{+\infty} \frac{P(x)}{Q(x)} \cos ax\,dx$



Here Jordan's Lemma will be used.

Type III. Indented path



this is for the case when there is a pole singularity on Real axis. If x_0 is a simple pole. then

$\lim_{\rho \to 0} \int_{C_\rho} f(z)\,dz = -\pi i\, a_{-1}$ Here a_{-1} is residue of f at x_0.

St The indented path can also be used to

evaluate $\int_0^{+\infty} \frac{x^{-a}}{x+1} dx$ for general a. (not necessarily integer. for this ~~Can~~ case, we need to define z^{-a} by branch cut. ∴ The origin should be cut out by a branch cut. general way to choose contour is