THE CHINESE UNIVERSITY OF HONG KONG
MATH 1540 Homework Set 3
Due time 6:30 pm Oct 31, 2016

1. Let $\mathcal{P}$ be the plane in $\mathbb{R}^3$ which contains the line:

   $$\vec{l}(t) = \langle 2t-1, 3, t\rangle, \quad t \in \mathbb{R},$$

   and the point $(2, -1, 1)$.
   Find an equation of the form $ax + by + cz = d$ which describes $\mathcal{P}$.

2. Consider the two planes $\mathcal{P}_1$ and $\mathcal{P}_2$ in $\mathbb{R}^3$, described respectively by the equations:

   $$x - y + 2z = 5,$$

   $$2x + 7y = 1.$$

   (a) Find a normal vector of length 1 of each of the two planes.
   (b) Find a vector parameterization of the line which is the intersection of the two planes.

3. Let L be the line in $\mathbb{R}^3$ described by the vector-valued function:

   $$\vec{l}(t) = \langle 1, -1, 7\rangle t + \langle 2, 0, 5\rangle, \quad t \in \mathbb{R}.$$

   Let $\mathcal{P}$ be the plane in $\mathbb{R}^3$ corresponding to the equation:

   $$4x - 3y - z = 3.$$

   Let $\mathcal{P}'$ be a plane which contains the origin, and whose intersection with $\mathcal{P}$ is the line L. Find an equation of the form $ax + by + cz = d$ which describes $\mathcal{P}'$.

4. Let L_1 and L_2 be two lines in $\mathbb{R}^3$ parameterized, respectively, by the following vector-valued functions:

   $$\vec{l_1}(t) = \langle t, 1+2t, -3-t\rangle, \quad t \in \mathbb{R};$$

   $$\vec{l_2}(t) = \langle -1+3t, 5t, 2t\rangle, \quad t \in \mathbb{R}.$$

   (a) Show that the two lines do not meet, and are not parallel to each other.
   (b) Find an equation whose graph is the plane containing L_2 and parallel to L_1.
   (c) Find the minimal distance between L_1 and L_2.

5. Show that the distance D between a point $P = (x', y', z')$ and the plane $ax + by + cz = d$ in $\mathbb{R}^3$ is given by:

   $$D = \left|\text{Proj}_{\vec{n}}\overrightarrow{P_0P}\right| = \frac{|ax' + by' + cz' - d|}{\sqrt{a^2 + b^2 + c^2}}$$

   (Here, $\vec{n}$ is any normal vector of the plane, and $P_0 = (x_0, y_0, z_0)$ is any point which lies on the plane.)